Idea Rents and Firm Growth

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Outline

Introduction

Empirical patterns

Model

- P/E ratios, $\bar{g}$

Quantification

- Calibration

Outline

This paper

Provides a new method for gauging firm contributions to growth using data on P/E ratios

P/E ratios contain information about the expected growth of firm rents

We build a tractable endogenous growth model in which idiosyncratic shocks to firm innovation step-sizes generate heterogeneous P/E ratios and growth in firm rents

We calibrate the model to publicly-listed U.S. firms to quantify the contribution of each firm to expected “aggregate” productivity growth

Motivation for new method

Patent and R&D data may miss some innovation

- Number of patents: Walmart 1/30 of Ford (Justia Patents 2020)
- Market cap: Walmart 10 times bigger than Ford (2024)
- Argente, Baslandze, Hanley, and Moreira (2023):
  - half of product innovation comes from firms that do not patent
  - larger firms have a much higher propensity to patent a new product
- Manufacturing and Information sectors: 85% of reported R&D vs 20% of value added (National Science Foundation BRDIS 2016, KLEMS)
- Larger firms more likely to report doing any R&D

Related literature

Gibrat's Law posits iid firm growth (embraced e.g. by Atkeson and Burstein, 2010)

- we find that larger firms contribute less to growth than Gibrat's Law would predict

Luttmer (2011): persistent shocks to the quality of a firm's new varieties $\Rightarrow$ faster growth

- in addition to the number of products, we highlight the role of markup heterogeneity
- and we quantify growth contributions based on the P/E ratios of publicly-listed firms

We estimate firm-level contributions to growth in all sectors without relying on patent or R&D data, which is scant outside manufacturing

Some of the existing evidence relating stock prices to innovation

- Pakes (1985 JPE) “On Patents, R&D, and the Stock Market Rate of Return”
- Berk, Green, and Naik (JoF 1999), “Optimal Investment, Growth Options, and Security Returns”
- Hall (2001 AER) “The Stock Market and Capital Accumulation”
- Vuolteenaho (2002 JoF) “What Drives Firm-Level Stock Returns”
- Hirschleifer, Hsu, and Li (2013 JFE) “Innovative Efficiency and Stock Returns”
- Kogan, Papanikolaou, Seru, and Stoffman (2017 QJE) “Technological Innovation, Resource Allocation, and Growth”
- Crouzet and Eberly (2023 JoF) “Rents and Intangible Capital: A Q+ Framework”
- Kalyani, Bloom, Carvalho, Hassan, Lerner, and Tahoun (2023 R&R QJE) “The Diffusion of New Technologies”

Empirically connecting P/E ratios to firm growth

P/E is the ex-dividend price divided by current earnings per share

$$\frac{P_0}{E_0} = \mathbb{E}\left[\sum_{t=1}^{\infty}\prod_{\tau=1}^{t}\frac{1+g_\tau}{1+r_\tau}\right]$$

where r_t is the discount rate and g_t is the growth rate of earnings between $t-1$ and t

If r and g are iid and independent of each other, we have the Gordon growth formula

$$\frac{P}{E} = \frac{1+\bar{g}}{\bar{r}-\bar{g}}$$

where $\bar{r} = \frac{1}{\mathbb{E}(1+r_t)^{-1}} - 1$ and $\bar{g} = \mathbb{E}(g_t)$

Empirically connecting P/E ratios to firm growth

$\bar{g}$ contains information about the growth of "rents" due to market power

- $\bar{g} = 0$ when Tobin's Q is 1 (such as in the neoclassical growth model)
- Atkeson, Heathcote, and Perri (2023 R&R AER)

More generally, given data on $r_{i,t}$ and $P_{i,t}/E_{i,t}$ ratios, we can calculate

$$\bar{g}_{i,t} \equiv \frac{r_{i,t} - \frac{1}{P_{i,t}/E_{i,t}}}{1 + \frac{1}{P_{i,t}/E_{i,t}}}$$

and construct a model to connect $\bar{g}$ to differences in innovation potential across firms

Benchmark: $r_{i,t} = \bar{r} \;\; \forall \, i, t$

Robustness: $r_{i,t}$ from David, Schmid and Zeke (JFE, 2022)

Example: Tesla vs. Ford

- the P/E ratio for Tesla was 13 times higher than that of Ford
- according to the David et al. measure, this was not explained by risk differences
- investors seem to expect much higher growth in rents for Tesla

	P/E ratio	implied $\bar{g}$ (%)
Ford	6.1	-6.0
Tesla	79.6	8.1

Note: 2023 P/E ratio using prices from CRSP and EPS from I/B/E/S.

$\bar{g} = (\bar{r} - \frac{E}{P})/(1 + \frac{E}{P})$. Both $\bar{r}$ and $\bar{g}$ are at annual frequency.

Example: Intel vs. Nvidia

	P/E ratio	implied $\bar{g}$ (%)
Intel	14.4	2.3
Nvidia	58.5	7.6

Note: Intel 12-2022 and Nvidia 01-2023. P/E ratio using prices from CRSP and EPS from I/B/E/S.

$\bar{g} = (\bar{r} - \frac{E}{P})/(1 + \frac{E}{P})$. Both $\bar{r}$ and $\bar{g}$ are at annual frequency.

Outline

Data sources

CRSP, Compustat, I/B/E/S, BEA

Baseline: set $\bar{r} = 9\%$ based on the market cap weighted average from David et al.

- Robustness: use the firm-level $r_{i,t}$ estimates from David et al.

Calculate firm-level expected earnings growth as $\bar{g}_{it} = \frac{\bar{r} - \frac{E_{it}}{P_{it}}}{1 + \frac{E_{it}}{P_{it}}}$

Sample covered

1976 – 2023

166,746 firm-year observations

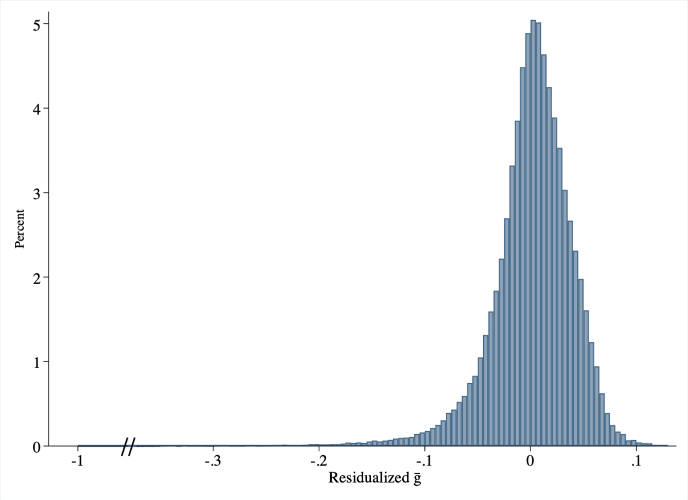
16,379 unique firms

3,474 firms/year on average

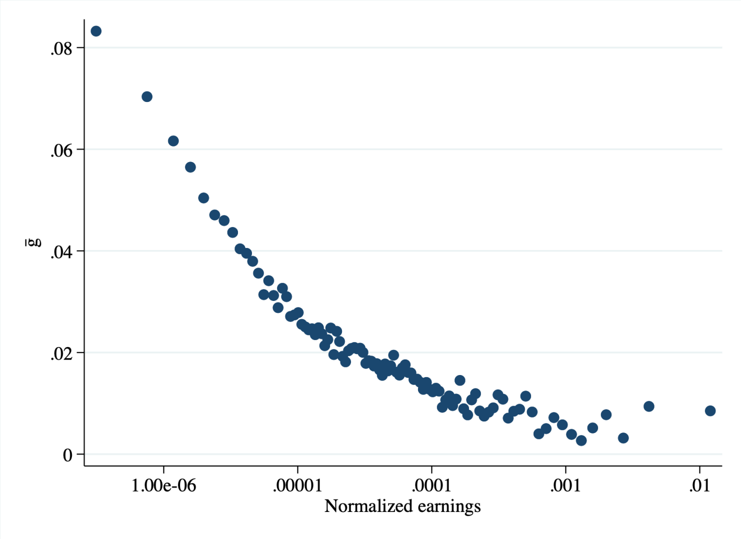
covers 1/3 of non-farm employment

Firms differ in their $\bar{g}$ values

Distribution of $\bar{g}$ across firms



Firms with higher earnings have lower $\bar{g}$ values



$\bar{g}$ predicts actual earnings growth

Growth forecast by $\bar{g}$ is strongly correlated with actual earnings growth

Regression of actual earnings growth k years ahead on current $\bar{g}$

k	1	2	3	4	5
Coefficient on $\bar{g}$	5.685	3.213	2.297	1.813	1.503
S.E.	(0.144)	(0.074)	(0.053)	(0.043)	(0.038)

$g_{E/S}$ explains 78% of the projection of $\bar{g}$ on actual earnings

Persistence of firm $\bar{g}$

Firm differences in $\bar{g}$ persist:

- AR(1) regression of $\bar{g}_{it}$ on $\bar{g}_{i,t-1}$ yields a coefficient of 0.55 (s.e. 0.009)
- AR(1) coefficient for the top half of the earnings distribution is 0.66 (s.e. 0.013)

Outline

Model outline

- Build on Klette and Kortum (2024)
  - Quality-driven growth
  - All innovation takes the form of creative destruction
  - Multi-product firms
- Ways we deviate from Klette-Kortum
  - Heterogeneous step size across firms
  - Shocks to firm step size
  - Arrival rate does not scale with a firm's number of products

Household

Representative household with preferences

$$U_0 = \sum_{t=0}^{\infty} \beta^t \log C_t$$

Final output is produced using a Cobb-Douglas aggregator of intermediate goods

$$Y = \exp\left(\int_0^1 \log\left[q(i)y(i)\right] di \right),$$

where $q(i)$ denotes the quality of good i and $y(i)$ its quantity

Intermediate goods production

Fixed number J of intermediate good producers

Each firm j has the knowledge to produce at quality $q(i,j)$ in line $i \in [0,1]$

Firm j can produce one unit of variety i at quality $q(i,j)$ using one unit of production labor:

$$y(i,j) = l(i,j)$$

Quality step sizes

When firm j innovates on line i where the current best quality is $q(i,j')$:

$$q(i,j) \;=\; \gamma_{jt} \cdot q(i,j')$$

γ_{jt} follows a Markov chain with two states $\gamma_H > \gamma_L > 1$ and transition probabilities

$$M = \begin{bmatrix} m_{HH} & m_{HL} \\ m_{LH} & m_{LL} \end{bmatrix}$$

If M is ergodic, then the stationary share of firms with γ_H is

$$s_H = \frac{m_{LH}}{1 + m_{LH} - m_{HH}}$$

Production costs

Quality-adjusted marginal cost for firm j to produce variety i is $w/q(i,j)$

In each product line i, $j(i)$ denotes the firm with the lowest (quality-adjusted) cost, and $j'(i)$ denotes the second-lowest (quality-adjusted) cost firm

Since $\gamma_S > 1$, $j(i)$ will be the producer with the highest quality in line i (see next slide)

Price setting

Bertrand competition within each product line $i \in [0,1]$

The leading firm sets its *quality-adjusted* price equal to the *quality-adjusted* marginal cost of the second-best producer:

$$\frac{p(i, j(i), j'(i))}{q(i, j(i))} = \frac{w}{q(i, j'(i))}$$

Markups

In line i, the leading firm $j(i)$'s choice of price implies the markup

$$\mu(i, j(i), j'(i)) \equiv \frac{p(i, j(i), j'(i))}{w} = \frac{q(i, j(i))}{q(i, j'(i))}$$

The price-cost markup on line i is endogenously determined by the quality of the best firm relative to the quality of the second-best firm

The markup in line i increases with the gap in quality between the leader and follower

Markup across lines under two states for the step size

The quality gap in a line depends on the step size of firm $j(i)$ when it took over the line:

$$\mu(i, j(i), j'(i)) = \begin{cases} \gamma_H & \text{if the step size was } \gamma_H \\ \\ \gamma_L & \text{if the step size was } \gamma_L \end{cases}$$

Line profits in each period

Due to the Cobb-Douglas aggregation of intermediates into final output, sales in each product line are given by Y and are independent of quality and price levels:

$$p(i) \cdot y(i) = Y, \quad \forall\, i.$$

And profit in a line is

$$Y\left(1 - \frac{1}{\mu(i, j(i), j'(i))}\right)$$

Firm profits in each period

Let n_j be the number of lines where firm j is the highest quality producer and let ω_j be the share of j's lines that have high markups

Firm j's total operating profit relative to output Y in a period is then

$$n_j \cdot \left[\omega_j \left(1 - \frac{1}{\gamma_H}\right) + (1 - \omega_j) \left(1 - \frac{1}{\gamma_L}\right)\right]$$

Innovation

Firms can gain lines by investing in R&D

Spending $\phi \cdot x_j^{\nu} \cdot Y$ in goods increases the best quality of x_j randomly drawn lines by $\gamma_{jt} > 1$

Period earnings relative to output Y of a firm with n lines who innovates on x lines is

$$E(n, \omega, x) \;=\; n\left[\omega\left(1 - \frac{1}{\gamma_H}\right) + (1-\omega)\left(1 - \frac{1}{\gamma_L}\right)\right] \;-\; \phi \cdot x^{\nu}$$

Firm innovation and value

Consider a stationary equilibrium with a constant aggregate rate of creative destruction x^*, a constant growth of Y at rate g^*, and a constant risk-free real interest rate r^*

Each period a firm loses $x^* \cdot n$ of its products to creative destruction

Given today's step size γ_{j0}, a firm chooses innovation to maximize the EPDV of profits:

$$V(n_{j0}, \omega_{j0}, \gamma_{j0}) = \max_{\{x_{jt}\}_{t=0}^{\infty}} \mathbb{E}_{\gamma_0} \sum_{t=0}^{\infty} E(n_{jt}, \omega_{jt}, x_{jt}) \left(\frac{1+g^*}{1+r^*}\right)^t$$

subject to

$$n_{j,t+1} = n_{jt} \cdot (1 - x^*) + x_{jt}$$

$$\omega_{j,t+1} n_{j,t+1} = (1 - x^*)\omega_{jt} n_{jt} + \mathbf{1}\{\gamma_{jt} = \gamma_H\} x_{jt}$$

R&D

The firm's dynamic problem has a recursive reformulation

$$V(n,\omega,\gamma) \;=\; \max_{x} E(n,\omega,x) + \frac{1+g^*}{1+r^*} \cdot \mathbb{E}[V(n',\omega',\gamma')|\gamma]$$

subject to

$$n' = n \cdot (1 - x^*) + x$$

$$\omega' n' = (1 - x^*)\omega n + \mathbf{1}\{\gamma = \gamma_H\} x$$

A firm's privately-optimal R&D increases with its current step size:

$$x(\gamma) = \left(\frac{1+g^*}{1+r^*} \frac{1}{\nu\phi} v(\gamma) \right)^{1/(\nu-1)}$$

where $v(\gamma) = \frac{(1+r^*)(1-1/\gamma)}{1+r^*-(1+g^*)(1-x^*)}$ is the PDV of a line relative to output Y

The growth rate on the BGP

We analyze the Balanced Growth Path (BGP) along which the size distribution of firms is stationary and aggregate quality grows at a constant rate

The aggregate rate of creative destruction is

$$x^* = \sum_j x_j = s_H \cdot x(\gamma_H) + (1 - s_H) \cdot x(\gamma_L)$$

The growth rate is the innovation-weighted geometric mean of the step sizes:

$$1 + g^* = (\gamma_H)^{x(\gamma_H)\cdot s_H} (\gamma_L)^{x(\gamma_L)\cdot(1-s_H)}$$

Closing the model

Final output can be used for consumption or to cover R&D costs:

$$Y = C + \sum_j \phi \cdot x_j^\nu \cdot Y$$

There is an exogenously fixed supply of 1 unit of production labor

We get the usual Euler equation for the representative household:

$$1 + g^* = \beta(1 + r^*)$$

Contribution to growth

A firm's size depends on the entire history of its step sizes

A firm's current innovation effort (R&D spending) depends only on its current step size

A firm's contribution to growth is independent of its size if step sizes are iid

A firm's contribution to growth is correlated with its size if step sizes are persistent

- can generate Gibrat's Law if states are highly persistent and similar

Using P/E ratios to infer step-size dynamics

The ex-dividend value of a firm is the expected PDV of future profits from its current portfolio of products *plus* the net expected value generated by its current and future R&D:

$$
\begin{aligned}
V(n,\omega,\gamma) &= n \cdot \left[\omega\left(1-\frac{1}{\gamma_H}\right)+(1-\omega)\left(1-\frac{1}{\gamma_L}\right)\right]\sum_{\tau=t}^{\infty}\left(\frac{(1+g^*)(1-x^*)}{1+r^*}\right)^{\tau-t} \\
&+ \left(1-\frac{1}{\nu}\right)\left(\frac{1}{\nu\phi}\right)^{1/(\nu-1)} \cdot \mathbb{E}_{\gamma}\sum_{\tau=t}^{\infty}\left(\frac{1+g^*}{1+r^*}\right)^{\tau-t}\left(\frac{1+g^*}{1+r^*}v(\gamma)\right)^{\nu/(\nu-1)} \\
&- E(n,\omega,\gamma)
\end{aligned}
$$

For given n and ω, the P/E ratio and $\bar{g}$ are higher for high step-size firms:

$$
\frac{P}{E}=\frac{V(n,\omega,\gamma)}{E(n,\omega,\gamma)}=\frac{V(n,\omega,\gamma)}{n\cdot\left[\omega\left(1-\frac{1}{\gamma_H}\right)+(1-\omega)\left(1-\frac{1}{\gamma_L}\right)\right]-\phi\left(\frac{1+g^*}{1+r^*}\frac{1}{\nu\phi}v(\gamma)\right)^{\nu/(\nu-1)}}
$$

Outline

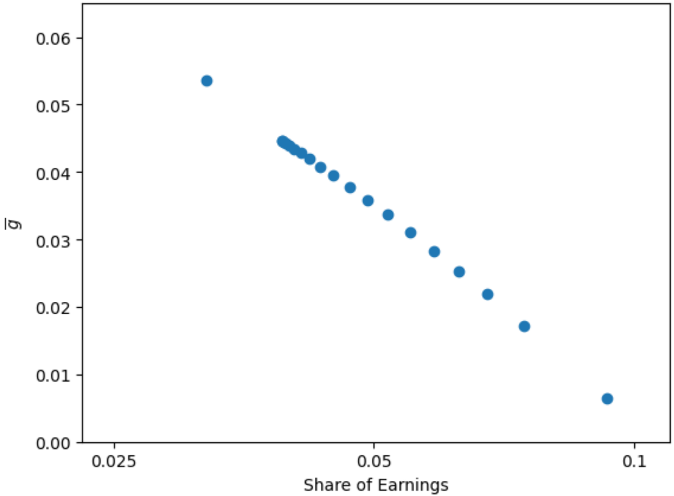
Calibration

Targets	Data	Model
1. Aggregate earnings growth net of labor input growth, g^*	4%	4%
2. Risk-free real interest rate, r^*	9%	9%
3. Contribution of earnings/sales to earnings growth	78%	85%
4. Minimum $\bar{g}_{it}$	0.7%	0.6%
5. Maximum $\bar{g}_{it}$	6.4%	5.4%
6. (Aggregate earnings)/(aggregate sales)	6.7%	6.7%
7. Persistence of $\bar{g}_{it}$	0.55	0.53

Parameters

Parameter		Value
1. Curvature of R&D cost function	ν	1.3
2. Scale of R&D cost function	ϕ	0.8
3. High step size	γ_H	1.24
4. Low step size	γ_L	1.19
5. Persistence of high state	m_{HH}	0.27
6. Persistence of low state	m_{LL}	0.92
7. Discount factor	β	0.95

Firms with higher earnings in the model have lower $\bar{g}$



Contributions to aggregate growth by firm size

- x-axis: share of sales by percentile of firm sales
- y-axis: contribution to growth

$$1\% \cdot \frac{x_H \cdot s_H^i \cdot \ln(\gamma_H) + x_L \cdot s_L^i \cdot \ln(\gamma_L)}{x_H \cdot s_H \ln(\gamma_H) + x_L \cdot s_L \ln(\gamma_L)}$$

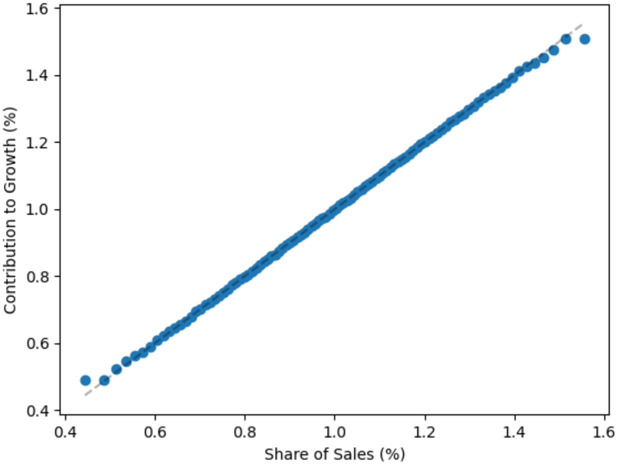
where s_k^i is the share for firms with step size k in bin i

Special cases:

- Gibrat's Law: contribution = % of sales $\rightarrow$ 45 degree line
- iid step size, $s_k^i = s_k \rightarrow$ contribution = 1% for all size percentiles

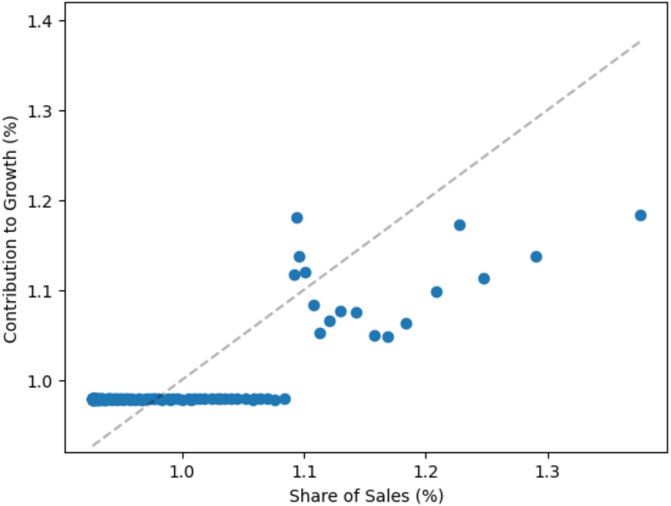
Contribution to growth if states are highly persistent and similar

Model can generate Gibrat's Law with persistent states $m_{LL} = m_{HH} = 0.85$ and similar step sizes $\gamma_H = 1.08$ and $\gamma_L = 1.05$



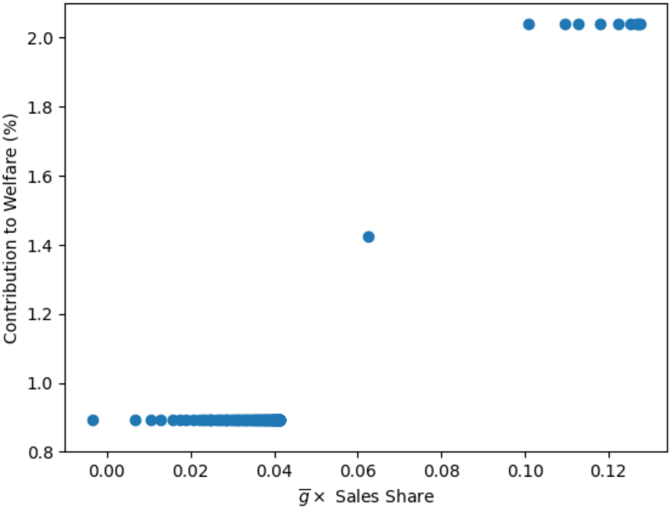
Contribution to growth by firm sales, calibrated model

Gibrat's Law overstates the contribution of large firms to growth



Contribution to growth over next year by $\bar{g} \times Sales$, calibrated model

Firms with higher $\bar{g} \times Sales$ contribute more to growth



Contribution to growth by the Magnificent Seven, 2023

Company	$\bar{g}\times$ Norm. Sales	Relative $\bar{g}$	Relative sales
Amazon	0.31	3.24	36.3
Apple	0.16	2.48	24.2
Alphabet	0.11	2.24	19.4
Microsoft	0.10	2.80	13.4
Tesla	0.06	3.55	6.12
Meta Platforms	0.05	2.21	8.53
NVIDIA	0.01	3.33	1.71

Conclusion

We propose a new method for inferring the contribution to aggregate productivity growth by individual firms using their P/E ratios

We construct a model with idiosyncratic and persistent (but not permanent) shocks to firm step sizes of innovation to interpret the data

Preliminary results suggest large firms contribute less to growth than their share of sales or rents would suggest

Can estimate the contribution of individual firms to expected growth — such as for the Magnificent Seven

To be done

Add more step-size states to generate “gazelles”

Let R&D cost fall with the firm’s number of products to generate more size dispersion

Model transitions from privately-held to publicly-listed firms

Adjust for empirical heterogeneity in leverage

How much are AI firms expected to contribute to productivity growth?