Idea Rents and Firm Growth

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Abstract

Which firms drive aggregate productivity growth? Price-earnings ratios differ markedly across publicly-listed firms. Large differences remain after netting out proxies for firm-specific discount factors. We find that high P/E firms tend to see increases in their earnings relative to sales, which we interpret as rents from ideas. We construct an endogenous growth model with persistent shocks to firm innovation step-sizes and calibrate it to match patterns in the data. The model implies that growth would be less than half as fast, even with the same innovative effort, if firms had the same step sizes. The model can be used to infer expected growth contributions of individual firms (such as members of the Magnificent Seven) and individual sectors (such as AI firms). We find that the share of growth coming from the smallest listed firms exceeds their 10% sales share, whereas the largest listed firms account for notably less than their 10% sales share.

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1 Introduction

The ratio of stock prices to earnings is well-known to be related to earnings growth prospects, as captured simply in the Gordon growth model equation $\frac{P}{E} = 1/(r - g)$, where r is the stochastic discount factor and g is the growth rate of earnings. To the extent that high P/E ratios predict high earnings growth, as found for example by Vuolteenaho (2002), such high P/E ratio may reflect heteregeneous "growth options" as in Berk, Green and Naik (1999) and Kogan and Papanikolaou (2014). Thus P/E ratios may shed light on the dynamics of firm innovation and how they relate to aggregate growth.

In this paper we do three things. First, we document patterns relating to firm-level P/E ratios using publicly-listed U.S. firms from 1976–2024. Second, we construct an endogenous growth model and calibrate it to match the empirical patterns. Third, we glean insights from our calibrated model about the contributions of different firms to aggregate productivity growth.

We confirm that high P/E ratios predict faster earnings growth in the cross-section of firms. Just as important, we find that high P/E ratios mostly predict faster growth in earnings *relative* to sales rather than simply parallel growth in sales and earnings. Firms with high earnings tend to have lower P/E ratios, and R&D-intensive firms tend to have higher P/E ratios.[1]

Motivated by the facts we document, we construct an endogenous growth model in the spirit of Klette and Kortum (2004). Like their model, ours features multiproduct firms engaging in creative destruction through quality innovations. We deviate in two ways that are critical for mimicking the data: (i) stochastic idiosyncratic innovation step sizes across firms, and (ii) innovation arrival rates that are a convex function of a firm's R&D expenditures alone.

Heterogeneous step sizes induce heterogeneity in firm R&D intensity, as price-cost markups and the profitability of innovation are increasing in step

[1] We also find that R&D intensity is declining in firm sales across firms within years. R&D coverage is much better within manufacturing than other sectors, so all of our results using R&D data are for manufacturing only.

sizes (Aghion and Howitt, 1992). Permanent differences in step sizes, however, would not generate heterogeneous P/E ratios in our model along a balanced growth path. Stochastic firm step sizes, in contrast, generate persistent but not permanent differences in P/E ratios across firms. High P/E ratio firms enjoy *temporarily* high step sizes, leading their sales to grow quickly for awhile. This is akin to Luttmer (2011), who concluded that temporarily good innovation opportunities were essential for understanding why many large U.S. firms ascended in just a few decades (e.g., Wal-Mart, Apple, Amazon). In our model, high P/E ratios portend faster growth in earnings *relative* to sales. As firms add high-markup products, they increase the average markup in their portfolio of products. On top of this, the firm's future R&D spending will revert back down along with its innovation step size, boosting earnings net of R&D.

Klette and Kortum (2004) assumed that R&D costs are increasing in a firm's innovation arrival rate, but decreasing in the firm's stock of products. We drop the stock of products from the firm R&D cost function, akin to Akcigit and Kerr (2018), which makes it harder for larger firms to innovate. Doing so renders the model more tractable as a firm's number of products are no longer a state variable affecting its R&D decision. More importantly, it helps us to explain why R&D/Sales ratios are falling in firm size among U.S. listed firms.

We calibrate our model to match six moments from the Compustat data: earnings growth per effective worker (a proxy for TFP growth); the average stochastic discount rate across firms in Compustat from David, Schmid and Zeke (2022); the share of predictable earnings growth from P/E ratios that is due to earnings/sales growth across firms; total earnings divided by total sales; the 90/10 ratio predicted earnings growth from P/E differences across firms; and the projection of R&D/Sales on log sales across manufacturing firms.

We choose values for six parameters to match the six moments: the scale and curvature of the R&D cost function (R&D spending as a function of the innovation arrival rate); the household's discount factor; plus the drift, persistence, and volatility of firm-specific innovation step sizes. The dispersion in innovation step sizes contributes to growth in that innovative firms are endogenously more research intensive. We find that growth would be 60% lower (0.6% rather than 1.5% per year) if aggregate innovative effort were the same but all firms shared the mean step size. Because large firms are less innovative and research intensive, we find that the largest firms, who collectively account for 10% of sales, contribute only 6% of aggregate TFP growth over the subsequent five years. Conversely, the smallest firms accounting for 10% of sales are responsible for 17% of TFP aggregate growth.

Related Literature

Our approach complements research using patents and R&D. This literature has uncovered rich insights, but patenting and R&D are less common outside manufacturing and software. Even within manufacturing, the propensity to patent may be low and vary across firms—see Argente, Baslandze, Hanley and Moreira (2020) for a comparison of products to patents in U.S. consumer packaged goods, and Aghion, Bergeaud, Boppart, Klenow and Li (2024) for the propensity to patent reported innovations in France. The methodology in the current paper can be applied to all firms with earnings and valuations, regardless of sector, patenting, or R&D reporting.

Like us, Kogan, Papanikolaou, Seru and Stoffman (2017) use stock prices to gauge the value of firm innovations. They examine changes in firm stock market capitalization in response to the announcement of firm patent grants. Our effort is distinct in not relying on patents and in focusing on the expected future growth contributions of firms rather than individual innovations. Our paper also relates to studies of Tobin's Q, such as Hall (1993), Hall (2001) and Crouzet and Eberly (2023). Tobin's Q has a backward-looking component (rents from past innovation), whereas the P/E ratio highlights the discounted value of future innovations that are not reflected in current earnings.

A vast literature explores the empirics of firm dynamics, in particular employment. Recent examples include Decker, Haltiwanger, Jarmin and

Miranda (2016), Garcia-Macia, Hsieh and Klenow (2019), Sterk, Sedláček and Pugsley (2021), and Klenow and Li (2021). Our approach emphasizes the need to look at earnings data as well, to get at innovation rates and hence growth contributions. We find that fast-growing firms, also known as gazelles and Luttmer Rockets, are associated with rising earnings relative to sales in addition to growing sales and employment.

Recent precursors to our quantitative modeling of firm dynamics and growth are Cao, Hyatt, Mukoyama and Sager (2017), Akcigit and Kerr (2018), Acemoglu, Akcigit, Alp, Bloom and Kerr (2018), Akcigit and Ates (2023), and Peters and Walsh (2025). Cao et al. model stochastic R&D efficiency (but not stochastic step sizes) in a model with variety creation and own-product innovation by incumbents. The middle three studies focus on firms who conduct R&D and who patent. Peters and Walsh aim to understand the impact of declining population growth on firm dynamism and productivity growth.

Campbell and Shiller (1988) and Cochrane (2011) decompose variation in stock market-wide P/E ratios into time variation in the stochastic discount factor versus growth in earnings (or cash flow). Similarly, Atkeson, Heathcote and Perri (2022) and Atkeson, Heathcote and Perri (2024) stress that fluctuating rents from innovation show up in aggregate P/E ratio movements. They explore the implications of this for time-varying U.S. stock market valuations and the U.S. net international asset position. In contrast to these studies, we investigate the implications of cross-firm dispersion in P/E ratios for firm growth contributions.

The rest of the paper is organized as follows. Section 2 documents empirical patterns surrounding firm P/E ratios and subsequent earnings growth. Section 3 builds an endogenous growth model to help interpret these patterns. Section 3 calibrates the model and characterizes how the implied firm growth contributions deviate from Gibrat's Law. Section 5 applies the methodology to estimating the expected future growth contributions of selected firms and sectors. Section 6 concludes.

2 Empirical patterns

2.1 Data source

Our main variables are stock prices from the Center for Research in Security Prices (CRSP), "street-level" earnings from the Institutional Brokers' Estimate System (I/B/E/S), and sales from Compustat. Our data cover 15,627 unique firms and 162,075 firm-year observations from 1976 to 2023. While the data covers only public firms, it accounts from roughly one-fourth of total private non-farm employment.

For the stock price, we source the security-level share price on the last trading date of each month from CRSP and aggregate to the firm level using outstanding shares. Our data covers securities listed on the NYSE, NYSE MKT, NASDAQ, and Arca exchanges. We extract security-level fiscal year-end earnings per share from I/B/E/S and aggregate to the firm-level using shares outstanding. The EPS is "street-level" in that I/B/E/S adjusts the accounting EPS to account for items that the majority of analysts include or exclude from their valuation calculations. We calculate the P/E ratio by dividing firm-level share price by firm-level earnings per share

$$\frac{P}{E} \equiv \frac{P}{EPS}$$

We also use firm-level earnings constructed by multiplying EPS by the number of shares outstanding

$$E = EPS \times \text{ number of shares outstanding}$$

Sales from Compustat is annual fiscal-year end sales from Compustat. We construct h year ahead the growth of earnings g_E^h and growth of earnings-to-sales ratio $g^h_{E/S}$ as

$$g_E = \frac{1}{h}\log\left(\frac{E_{t+h}}{E_h}\right) \qquad g_{E/S} = \frac{1}{h}\log\left(\frac{E_{t+h}/S_{t+h}}{E_t/S_t}\right)$$

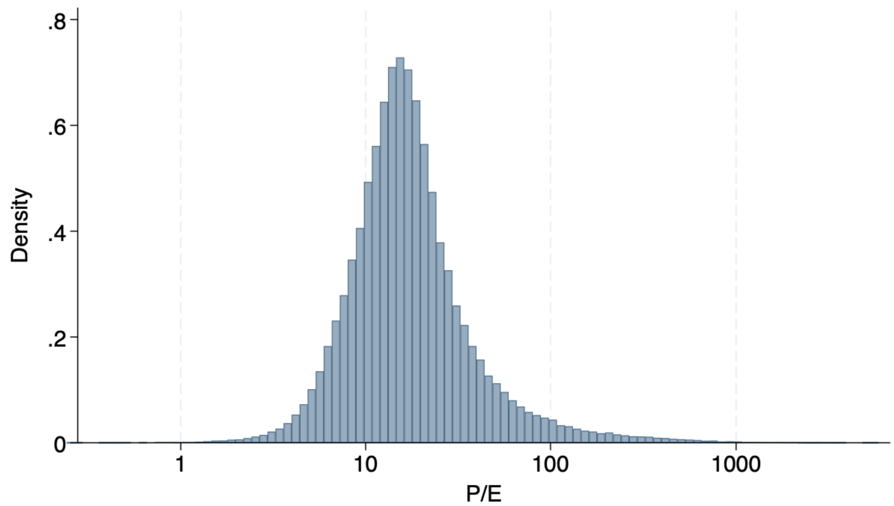
We drop firms with negative earnings when calculating growth of earnings. We also drop negative earnings in the model when we compare earnings growth in the model with the data.

2.2 Motivating facts

Significant dispersion in P/E Figure 1 shows the distribution of raw P/E ratios pooled across firm-years. Much of this variation is across firms as opposed to across years (i.e., it is not just common time effects). Figure 2 shows the standard deviation and IQ range across firms within each year of our sample. The Appendix Figures A1 shows significant dispersion after taking out dispersion in year fixed effects and fiscal-year end month fixed effects. The cross-sectional variation is both across industries and across firms within industries as show in Appendix Figure A2.

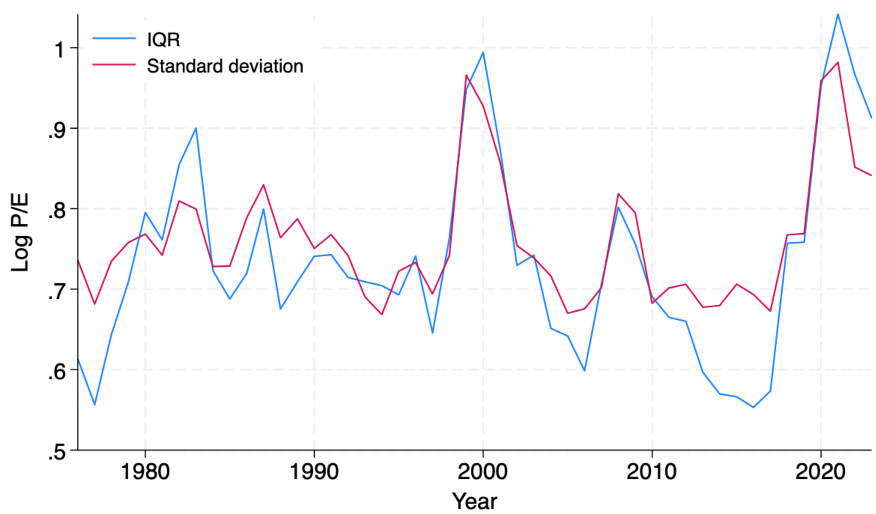
The cross-sectional dispersion is not just about dispersion in firm-specific risks. We calculate the residual of $\frac{P}{E}_{it} - \frac{1}{r_{it}}$ for each firm i and year t, where r_{it} is the firm-specific risk measured by David et al. (2022) using Fama-French factor models. According to the Gordon growth formula, the residual captures variation in expected growth of rents. We find significant dispersion in the residual (see Appendix Figure A3). This is consistent with the finding of Vuolteenaho (2002) that differences in firm-level stock returns is predominantly driven by changes in cash-flow expectations as opposed to changes in the discount rate.

Figure 1: Distribution of P/E



Source: CRSP and I/B/E/S. Figure trims the top and bottom 0.1% observations each year.

Figure 2: P/E over time

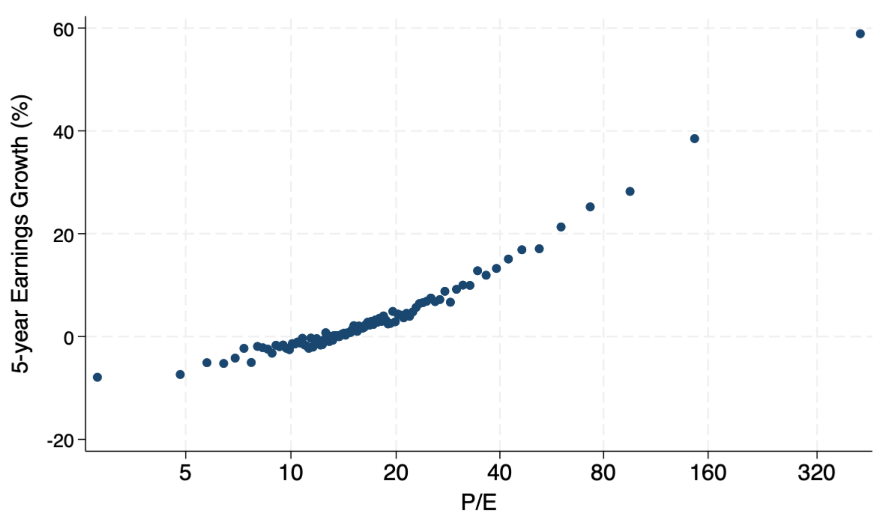


Source: CRSP and I/B/E/S.

High P/E firms have higher earnings growth and R&D intensity One may be concerned that the dispersion in P/E is simply noise. However, this is not the case. Figure 3 plots the 5-year ahead earnings growth against P/E. High P/E ratio today predicts high subsequent earnings growth. In addition to earnings growth, high P/E firms also have higher R&D intensity, as shown in Figure 4.

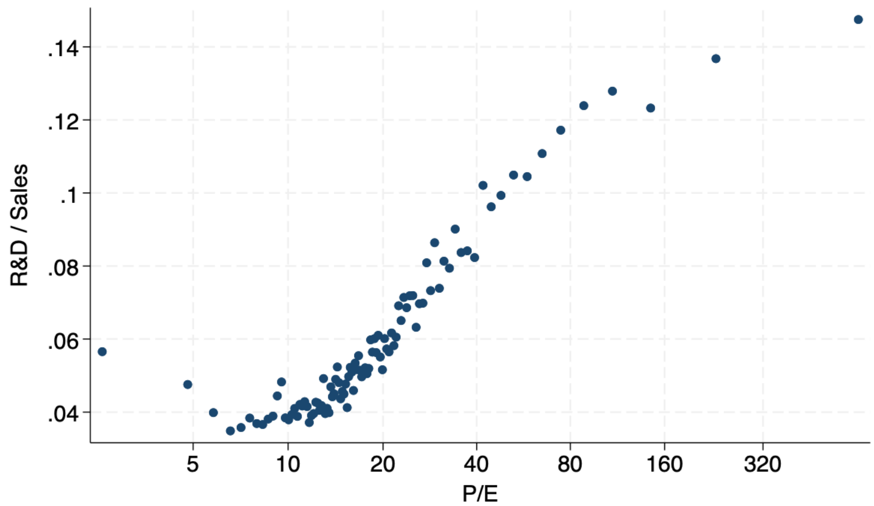
Markups may play an important role in the relationship between earnings growth and P/E ratios. Row 1 of Table 1 shows the coefficient of regressing h-year ahead earnings growth for $h = 1, 2, \ldots, 5$. At all horizons, higher P/E ratios predicts higher earnings growth. Row 2 and 3 decomposes this relationship into growth of earnings-to-sales ratio and sales. The bulk of the relationship between earnings growth and P/E is account for by higher P/E ratio predicting higher growth in earnings-to-sales.

Figure 3: Earnings growth vs P/E



Source: Compustat. The y-axis is average yearly earnings growth from year t to $t + 5$. The x-axis is P/E ratio in t.

Figure 4: R&D/Sales vs P/E



Source: Compustat. The y-axis is R&D over sales in year t. The x-axis is P/E ratio in t.

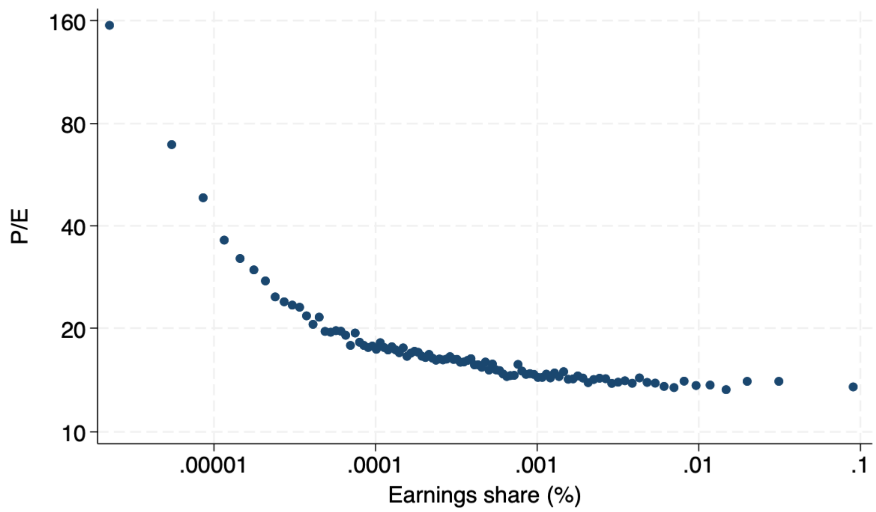
Table 1: Growth of earnings and earnings/sales vs P/E ratio

	$h=1$	$h=2$	$h=3$	$h=4$	$h=5$
g_E on $\log(P/E)$	0.422	0.262	0.189	0.150	0.127
	(0.005)	(0.003)	(0.002)	(0.002)	(0.002)
$g_{E/S}$	0.355	0.213	0.151	0.117	0.098
	(0.005)	(0.003)	(0.003)	(0.002)	(0.002)
g_S	0.067	0.049	0.039	0.033	0.029
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)

Source: Compustat. The first row displays the coefficient when regressing average yearly earnings growth from year t to $t+h$ on P/E ratio in t. The second and third row decomposes the coefficient into growth in the ratio of earnings to sales and sales.

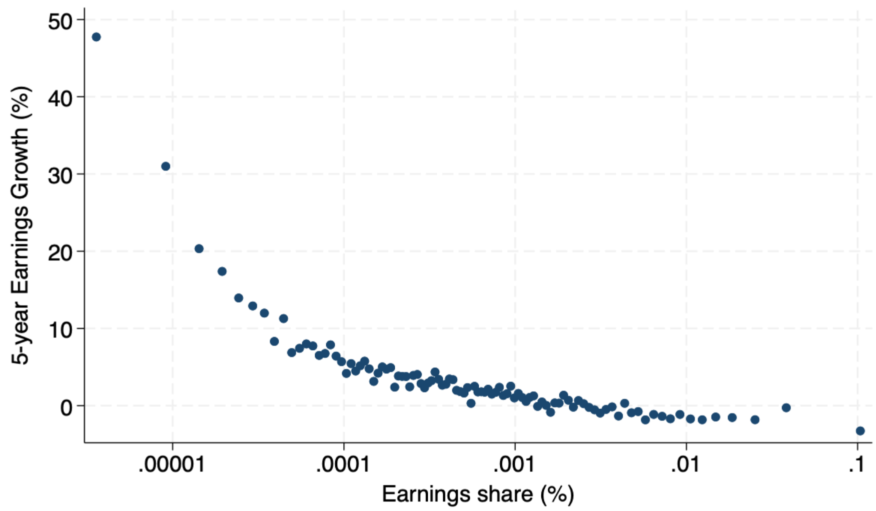
Firms with higher earnings have lower P/E ratio and earnings growth While high P/E ratio predicts high earnings growth, high earnings firms have lower P/E ratio and earnings growth. Figure 5 plots P/E of a firm against its share of total earnings and Figure 5 plots the 5-year ahead earnings growth of a firm against its share of total earnings. Both the P/E ratio and earnings growth decline with earnings. Appendix Figure A4 shows that sales growth also declines with sales. Table 2 shows that at all horizons the negative relationship between earnings growth and earnings is predominantly through a negative relationship between growth in E/S and current earnings.

Figure 5: P/E ratio vs. earnings



Source: Compustat. The y-axis is P/E ratio in year t. The x-axis is share of total earnings in t.

Figure 6: Earnings growth vs earnings



Source: Compustat. The y-axis is average yearly growth of earnings from year t to $t + 5$. The x-axis is share of total earnings in t.

Table 2: Growth of earnings and earnings/sales vs earnings

	$h = 1$	$h = 2$	$h = 3$	$h = 4$	$h = 5$
g_E on $\log(E)$	-0.066	-0.047	-0.036	-0.031	-0.027
	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)
$g_{E/S}$	-0.051	-0.033	-0.024	-0.020	-0.017
	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)
g_S	-0.016	-0.014	-0.012	-0.011	-0.011
	(0.0005)	(0.0004)	(0.0004)	(0.0004)	(0.0004)

Source: Compustat. The first row displays the coefficient when regressing average yearly earnings growth from year t to $t + h$ on earnings in t. The second and third row decomposes the coefficient into growth in the ratio of earnings to sales and sales.

3 Model

Based on the facts we documented in the previous section, in this section we formulate an endogenous growth model with persistent and predictable (but not permanent) differences in innovation step sizes across firms. Growth in earnings/sales (especially earnings gross of R&D expenditures) is suggestive of rising markups at high P/E firms. Thus perhaps high P/E ratio firms have opportunities to invest in innovations with high quality steps that will command high markups, fueling their fast earnings growth. A high P/E ratio could then suggest that investors have information about the firm's investment opportunities that they price into firm valuations. If firms have information about their opportunities as well, this could account for the high R&D intensity of high P/E ratio firms.

Thus, the model builds on Klette and Kortum (2004), which features endogenous quality growth and creative destruction by multiproduct firms. The key difference is that we allow each firm's innovation step size to evolve stochastically (and idiosyncratically) rather than being common or fixed over time. This will create markup dynamics and in turn heterogeneous P/E ratios to help the model match the patterns we found in the data.

3.1 Set up

Household A representative household inelastically supplies L units of production labor and maximizes its discounted utility

$$\max_{\{C_t\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log(C_t) \tag{1}$$

subject to

$$A_{t+1} = A_t(1 + r_t) + w_t L - C_t, \tag{2}$$

and a standard no-Ponzi game condition. Here C denotes consumption and A the household's wealth from owning all of the firms in the economy. Here r denotes the interest rate, w is the wage rate of production labor and $\beta \in (0,1)$ the discount factor. The household's Euler equation links the equilibrium growth rate and interest rate to the discount factor as follows

$$1 + g = \beta(1 + r). \tag{3}$$

Final goods production Final output, Y is a Cobb-Douglas bundle of a unit interval of intermediate goods which come at qualities $q(i)$

$$Y = \exp\left(\int_0^1 \log\left(q(i)y(i)\right) di\right).$$

The final output good can be used consumption C or R&D R.

We assume the final goods market is competitive. So final output production can be characterized as the behavior of a representative firm solving

$$\max_{\{y(i)\}_{i=0}^1} P \cdot \exp\left(\int_0^1 \log\left(q(i)y(i)\right) di\right) - \int_0^1 p(i)y(i)\, di. \tag{4}$$

taking prices P and $p(i)$ as given. Due to the Cobb-Douglas structure, the solution to the final goods producer's problem features equal spending on each product

$$p(i)y(i) = PY. \tag{5}$$

In the following, we normalize the price of the final output $P = \exp\left(\int_0^1 \log\left(p(i)/q(i)\right) di\right)$ to 1 in all periods.

Intermediate goods production The intermediate goods are produced by a large fixed J number of firms indexed by $j \in \{1, \cdots, J\}$. Each firm j has a blue print to produce each variety i with quality $q(i,j)$. The firms needs one unit of production labor to produce one unit of the variety. More specifically, firm j

can produce one unit of good i at quality $q(i, j)$ with technology $y(i, j) = l(i, j)$ where $l(i, j)$ is the labor input of firm j into producing good i.

The quality levels at which a firm produces change endogenously over time as a result of R&D activity. Each firm has access to a R&D technology with heterogeneous and stochastic step sizes. That is, they can randomly draw x number of lines to innovate upon by spending

$$\phi x^{\nu} Y \tag{6}$$

units of final output. The firms can also loose products at rate $\bar{x}$ that is endogenous but taken as given by the firms.

For the lines drawn by firm j in period t, the firms can improve the highest quality in the line by a factor of γ_{jt}. The step size of a firm γ_{jt} is stochastic and follows a N-state Markov chain. It can take on N values: γ_k, $k \in \{1, 2, ..., N\}$ with $\gamma_{k+1} > \gamma_k \geq 1$. The transition probability is given by

$$M = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1N} \\ \vdots & \vdots & \vdots & \vdots \\ m_{N1} & m_{N2} & \cdot & m_{NN} \end{bmatrix} \tag{7}$$

where $m_{kk'}$ denotes the probability of a firm with step size γ_k in t moving to step size $\gamma_{k'}$ in $t + 1$. We assume the Markov chain is ergodic.

We assume the intermediate inputs market features Bertrand competition. Under this, the producer in each line is the highest quality producer who sets the price to the quality adjust marginal cost of the second highest quality producer $p(i) = \frac{w}{q(i,j'(i))}$ while its own marginal cost is $\frac{w}{q(i,j(i))}$. Hence markup in each line is determined by the relative quality of the best and second best producers

$$\mu(i) = \frac{q(i, j(i))}{q(i, j'(i))} \tag{8}$$

Since the highest quality producer is the firm that last innovated on the line, the markup in a line is the quality step size of the best producer *at the time it acquired the line.* This is in general different from the current step size of the producer. Going forward, we will use $\gamma(i)$ to denote the markup on the line i. Thus the profit of the firm producing in line i relative to aggregate output Y is

$$\pi(\gamma(i)) = \left(1 - \frac{1}{\gamma_i}\right) \tag{9}$$

Let $\mathcal{N}_j$ be the portfolio of lines where firm j is the highest quality producer. The measure of $\mathcal{N}_j$ is n_j. Firm j's total earnings before expensing R&D relative to output Y in a period is

$$\int_{i\in\mathcal{N}_j} \pi(\gamma(i))\, di.$$

Since there are N types of step size, we can write the earnings before R&D as

$$\tilde{E}(\{\omega_k^j\}, n_j) = n_j \sum_k \omega_k^j \pi(\gamma_k) \tag{10}$$

where ω_k^j is the share of firm j's portfolio where the firm had step size γ_k when it acquired the line and n_j is the number of lines in the portfolio. Earnings after R&D is

$$E(\{\omega_k^j\}, n_j, x_j) = \tilde{E}(\{\omega_k^j\}, n_j) - \phi x_j^{\nu}. \tag{11}$$

In each period, the firm sees its current step size γ_{jt} and then chooses the number of lines x_j to draw to maximize the present discounted value of the firm. The value of the firm (before dividend) is the present discounted value of its earnings stream

$$V_{jt} = \max_{\{x_{j,\tau}, n_{j,\tau+1}\}_{\tau=t}^{\infty}} \sum_{\tau=0}^{\infty} \left(\prod_{s=1}^{\tau} \frac{1}{1 + r_{t+s}} \right) E(\{\omega_{k,t+\tau}^j\}, n_{j,t+\tau}, x_{j,t+\tau}) Y_{t+\tau}, \tag{12}$$

subject to

$$n_{\tau+1} = n_\tau(1 - \bar{x}) + x_\tau,\ \forall\tau.$$

The earnings stream in turn depends on the history of the firm's step size draws, how fast the firm looses its products to creative destruction and its innovation decisions

$$E(\{\omega^j_{k,t+1}\}, n_{j,t+1}, x_{j,t+1}) = \tilde{E}(\{\omega^j_{kt}\}, n_{jt})(1-\bar{x}_t) + x_{jt}\pi(\gamma_{jt}) - \phi x^{\nu}_{j,t+1}. \tag{13}$$

We consider a stationary equilibrium on which the interest rate r, the rate of creative destruction $\bar{x}$ and the growth rate of $g_t = Y_t/Y_{t-1} - 1 = g$ are constant $\forall t$. Then the problem of the firm simplifies to solving

$$\begin{aligned} V(\{\omega^j_{kt}\}, n_{jt}, \gamma_{jt}) &= \tilde{E}(\{\omega^j_{kt}\}, n_{jt}) \frac{1+r}{1+r-(1+g)(1-\bar{x})} \qquad (14) \\ &+ \max_{\{x_{j\tau}\}^{\infty}_{\tau=t}} \mathbb{E}_{\gamma_{jt}} \sum_{\tau=0}^{\infty} \left(\frac{1+g}{1+r}\right)^{\tau} \left(x_{j,t+\tau} v(\gamma_{j,t+\tau}) \frac{1+g}{1+r} - \phi x^{\nu}_{j,t+\tau}\right). \end{aligned}$$

Here $v(\gamma)$ denotes the present discounted value relative to output Y of a line with step size γ between the best and second best producer

$$v(\gamma_i) = \pi(\gamma_i) \frac{1+r}{1+r-(1+g)(1-\bar{x})}. \tag{15}$$

From the above problem, the optimal choice of x_{jt} only depends on γ_{jt}. Hence we can write the R&D decision as choosing a policy function $x(\gamma)$ that solves

$$\max_{x \geq 0} x v(\gamma) \frac{1+g}{1+r} - \phi x^{\nu}. \tag{16}$$

The solution to this is

$$x(\gamma) = \left(\frac{1+g}{1+r} v(\gamma) \frac{1}{\nu\phi}\right)^{1/(\nu-1)} \tag{17}$$

and the additional firm value generated by today's R&D net of the R&D cost is

$$\left(\frac{1+g}{1+r} v(\gamma)\right)^{\nu/(\nu-1)} \cdot \left(1 - \frac{1}{\nu}\right) \left(\frac{1}{\nu\phi}\right)^{1/(\nu-1)} \tag{18}$$

Hence, firms with higher step sizes spend more on R&D and add more value.

Substitute the optimal R&D decision into the firm's value function yields the value of firm

$$V(\{\omega_{kt}^j\}, n_{jt}, \gamma_{jt}) = \tilde{E}(\{\omega_{kt}^j\}, n_{jt}) \sum_{\tau=t}^{\infty} \left(\frac{(1+g)(1-\bar{x})}{1+r} \right)^{\tau-t} + \left(1-\frac{1}{\nu}\right)\left(\frac{1}{\nu\phi}\right)^{1/(\nu-1)} \cdot \mathbb{E}_{\gamma_{jt}} \sum_{\tau=0}^{\infty} \left(\frac{1+g}{1+r}\right)^{\tau} \left(\frac{1+g}{1+r} \upsilon(\gamma_{j,t+\tau})\right)^{\nu/(\nu-1)}. \quad (19)$$

The price to earnings ratio is then

$$\frac{V(\{\omega_{kt}^j\}, n_{jt}, \gamma_{jt})}{\tilde{E}(\{\omega_{kt}^j\}, n_{jt})} = \frac{1+r}{1+r-(1+g)(1-\bar{x})} + \left(1-\frac{1}{\nu}\right)\left(\frac{1}{\nu\phi}\right)^{1/(\nu-1)} \cdot \mathbb{E}_{\gamma_{jt}} \frac{\sum_{\tau=0}^{\infty} \left(\frac{1+g}{1+r}\right)^{\tau} \left(\frac{1+g}{1+r} \upsilon(\gamma_{j,t+\tau})\right)^{\nu/(\nu-1)}}{\tilde{E}(\{\omega_{kt}^j\}, n_{jt})}.$$

The first term does not depend on firm size. The second term depends on the stochastic process of the step size. When step sizes are iid, this term declines with today's earnings.

Market clearing and aggregates Since we have a unit measure of lines, the aggregate rate of creative destruction is equal to

$$\bar{x} = \sum_{j=1}^{J} x(\gamma_{jt}) = \sum_{k=1}^{N} J s_k x(\gamma_k), \quad (20)$$

where s_k is the stationary distribution of step sizes according the the Markov transition probabilities M. It is the share of firms with step size γ_k in each period.

The equilibrium growth rate of aggregate output (holding fixed aggregate labor input) is

$$1+g = \prod_{k=1}^{N} (\gamma_k)^{x(\gamma_k)\cdot s_k J}. \quad (21)$$

Equations (20), (21), and the household's Euler equation (3) jointly pins down equilibrium $\bar{x}$, r and g.

We also have the labor market clearing condition

$$\int_0^1 l(i,j)di = L,$$

and the asset market clearing condition

$$\sum_{j=1}^{J} V_{jt} = A_t.$$

Finally, we have the number of lines operated by the firms summing to 1

$$1 = \sum_{j=1}^{J} n_j.$$

Equilibrium definition, BGP definition A decentralized equilibrium consists of a sequence of quantities and prices that jointly solve the final producer's problem, the intermediate producers' problems, the household's problem and is consistent with the market clearing conditions and all the aggregate constraints. As mentioned above, we define a balanced growth path (BGP) in the standard way, i.e., as a path along which all quantities grow at constant rates.

4 Calibration and consequences

In this section, we calibrate the model and calculated the growth contribution of firms. To keep the model parsimonious we approximate the Markov chain in the model with an AR(1) process using the Rouwenhorst method. Namely, we calibrate the persistent parameter ρ and volatility parameter σ_ε of process

$$\log(\gamma_{jt} - 1) = \rho \log(\gamma_{j,t-1} - 1) + \varepsilon_t, \quad \varepsilon \sim N(\mu_\varepsilon, \sigma_\varepsilon^2).$$

We use the Rouwenhorst method to generate $N = 20$ discrete states $\log(\gamma_{kt} - 1), \ldots, \log(\gamma_{Nt} - 1)$ and the associated transition matrix. We use this for the Markov process of the step size in the model.

We calibrate 6 parameters to 6 moments. The parameters are the curvature ν and scale ϕ of R&D cost function, the drift μ_ε, persistence ρ and volatility σ_ε of the step size AR(1) process, and the discount factor β. The six moments are the growth of average earnings net of labor input growth, the average stochastic discount factor from David et al. (2022), the contribution of earnings-to-sales ratio to the relationship between earnings growth and P/E, aggregate earnings to sales ratio, the dispersion of earnings growth, and the relationship between R&D intensity and sales.

The aggregate earnings/sales and growth in average earnings together are informative of the drift μ_ε and the scale ϕ of R&D cost function. The dispersion in earnings growth is informative of the volatility parameter σ_ε. Given the aggregate growth moment, the discount rate pins down β. The contribution of earnings/sales is informative of the curvature parameter. When curvature is high, firms R&D choice and hence the number of products they innovate upon does not grow as much with the step size draw. As a result, a firm's earnings growth comes less from variation in the number of product lines it adds and more from growth in earnings/sales via different step sizes. The R&D intensity relationship with sales is informative of the persistence parameter ρ. All else equal, higher ρ generates more dispersed sales distribution and flattens the slope.

Table 3 displays the target and data moments while Table 4 displays the calibrated parameters. The overall fit is good, although we generate less aggregate earnings/sales than in the data. The implied cost weighted price-cost markup is 1.24 in our calibrated economy. We find that the step sizes are not persistent with $\rho = 0.22$.

Table 3: Calibration data vs model

Targets	Data	Model
1. Growth in average earnings, net of labor input growth, $g^{\star}$	1.3%	1.5%
2. Average stochastic discount factor, $r^{\star}$	9.7%	9.9%
3. Contribution of earnings/sales to earnings growth	70%	50%
4. Aggregate earnings/sales	0.28	0.19
5. 90-10 range of fitted earning growth	0.15	0.12
6. R&D/sales projected on sales	-0.028	-0.026

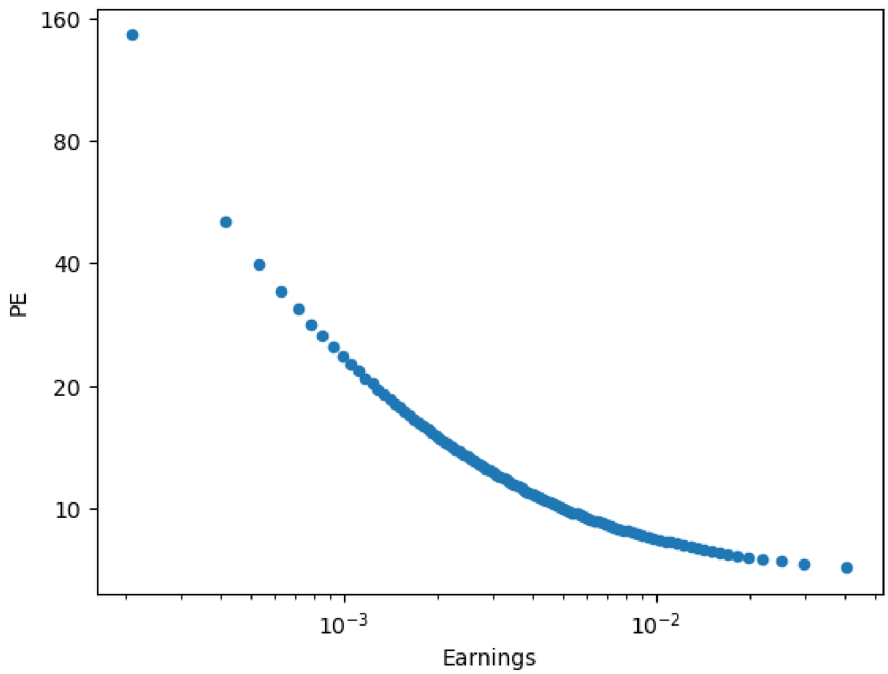
Source: 1. earnings from I/B/E/S and labor input growth from the BLS. 2. average of firm-specific discount rate from David et al. (2022). 3 to 6 are from CRSP, I/B/E/S, and Compustat.

Table 4: Calibrated parameters

Parameter		Value
Curvature of R&D cost function	ν	2.04
Scale of R&D cost function	ϕ	1.77
Persistence of γ process	ρ	0.22
Mean of shocks to γ process	μ_{ε}	-4.10
Variance of shocks to γ process	σ_{ε}^{2}	2.00
Discount factor	β	0.92

The calibrated model is able to rationalize the decline of P/E ratio with earnings across firms. Figure 7 shows the relationship between P/E ratio and earnings. According to our model, the P/E is a constant plus the expected PDV of future earnings over current earnings. Current earnings depend on the history of step size shocks while future earnings depends on the current step size. When step sizes are not persistent, current step size is not highly correlated with earnings, leading to a decline in P/E with earnings.

Figure 7: P/E vs. earnings, model



4.1 Firm's contribution to growth

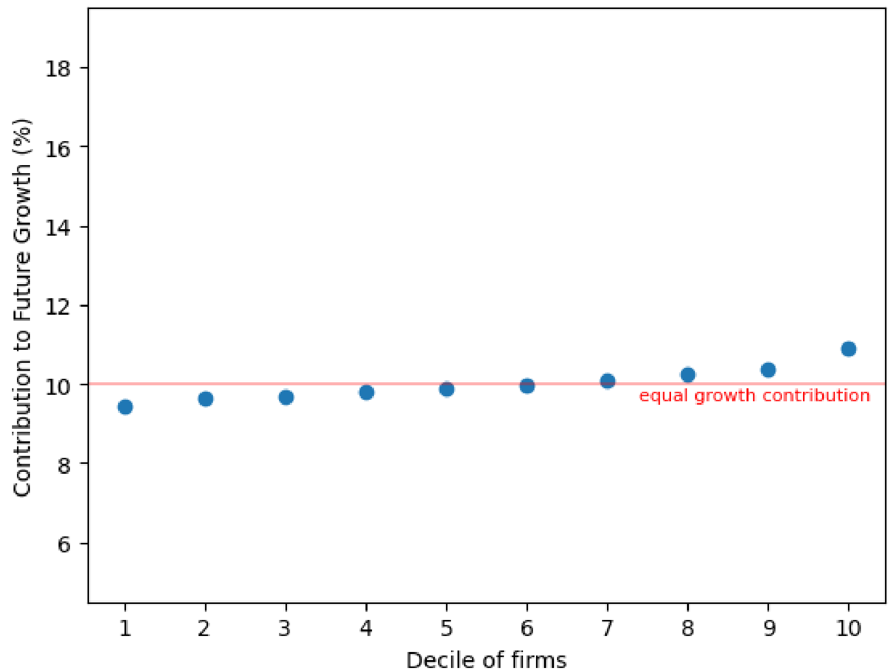
Using the calibrated model, we can calculate firms contribution to growth. For a group of firms I, contribution to growth is

$$m(I) \cdot \frac{\sum_k x(\gamma_k) \cdot s_k^i \cdot \ln(\gamma_k)}{\sum_k x(\gamma_k) \cdot s_k \cdot \ln(\gamma_k)}$$

where s_k^i is the share for firms with step size k and $m(I)$ is the fraction of firms in I. The so-called Gibrat's Law refers to the special case when contribution to growth is equal to the sales share of the firms. In contrast, when step sizes are independent, the contribution is 1% for all size percentiles.

Using this metric, we find that large firms (higher sales) do contribute more to growth but not as much as what Gibrat's Law predicts. Figure 8 ranks the firms by sales and show the contribution of firms in each decile of firms. The largest 10% of firms contributes to more than 10% of growth. Figure 9 shows

Figure 8: Contribution to growth by firm decile



the contribution by sales deciles. The largest firms that accounts for 10% of sales share contributes to less than 10% of growth. This is the result of a step size process that is mildly persistent.

5 Application

TBD

Our calibrated model suggests that the P/E ratio is a good indicator of firm's contribution to aggregate growth. Figure 10 plots the contribution to growth against the P/E ratio. Firms with higher P/E ratio contributes more to growth.

Figure 9: Contribution to growth by sales decile

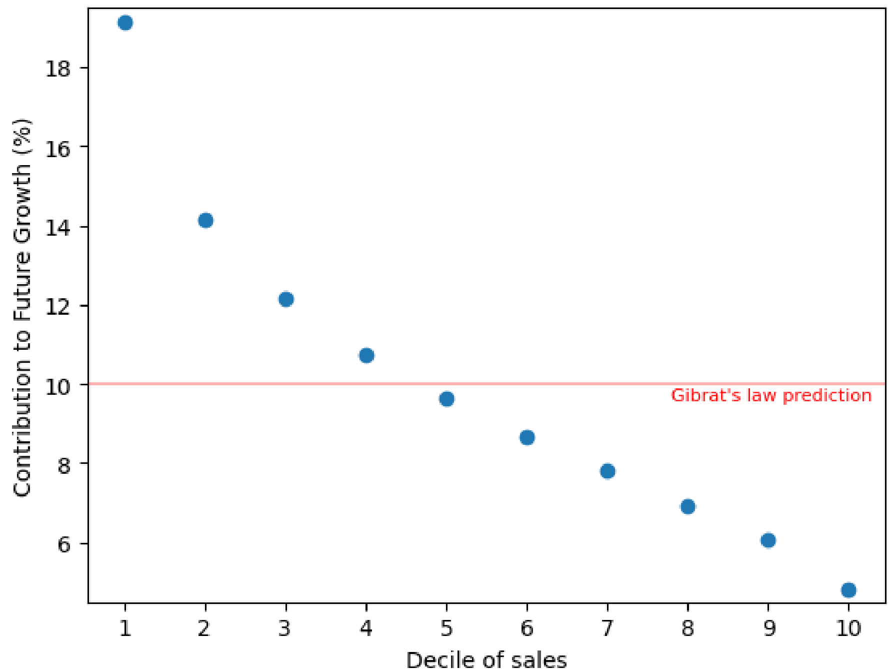
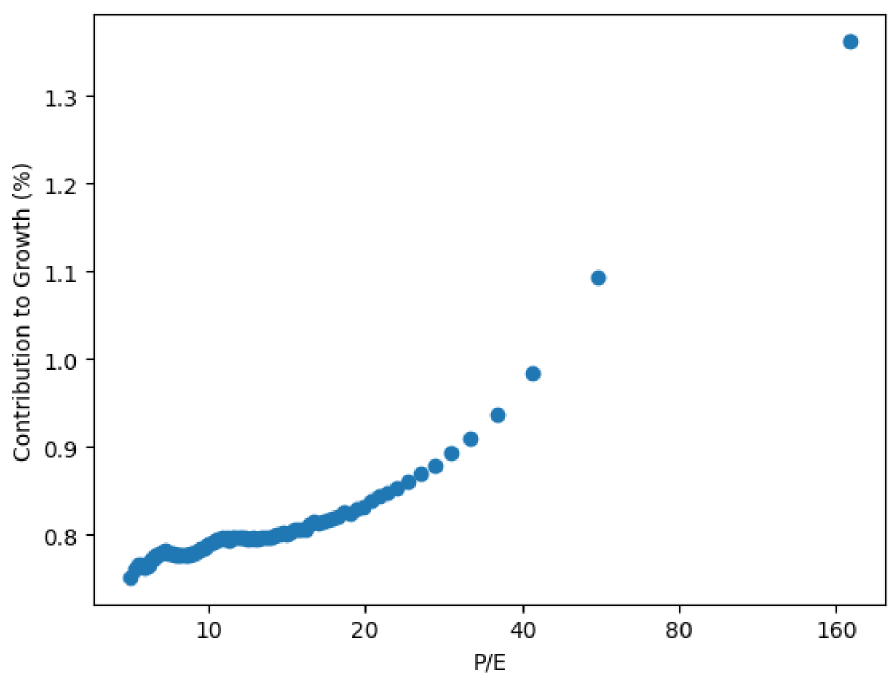


Figure 10: Contribution to growth by P/E



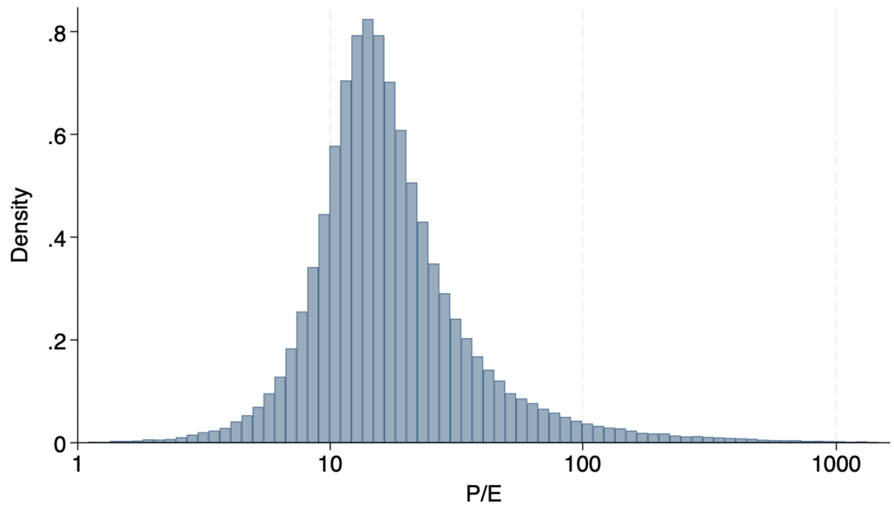
6 Conclusion

This paper constructs an endogenous growth model with persistent shocks to firm innovation step-size and calibrates it to match the relationship between P/E ratios, earnings, earnings growth, and earnings-to-sales growth across firms. Shocks to the step-size of firms generates differences in future earnings and hence differences in valuation relative to current earnings. Our calibrate model implies significant deviation from the Gibrat's Law with the largest firms contributing less than their share of sales.

Our model suggests that the P/E ratio is a good indicator of firm's contribution to growth. There are potential policy uses for real-time estimates of firm contributions to future productivity growth. Because innovations build on each other, large step-size firms confer larger knowledge spillovers. Aghion, Bergeaud, Boppart, Klenow and Li (2024) highlight potential growth gains from reallocation research effort from low to high step-size firms. If governments offer enhanced research subsidies, however, then firms might have an incentive to manipulate their market capitalization or earnings.

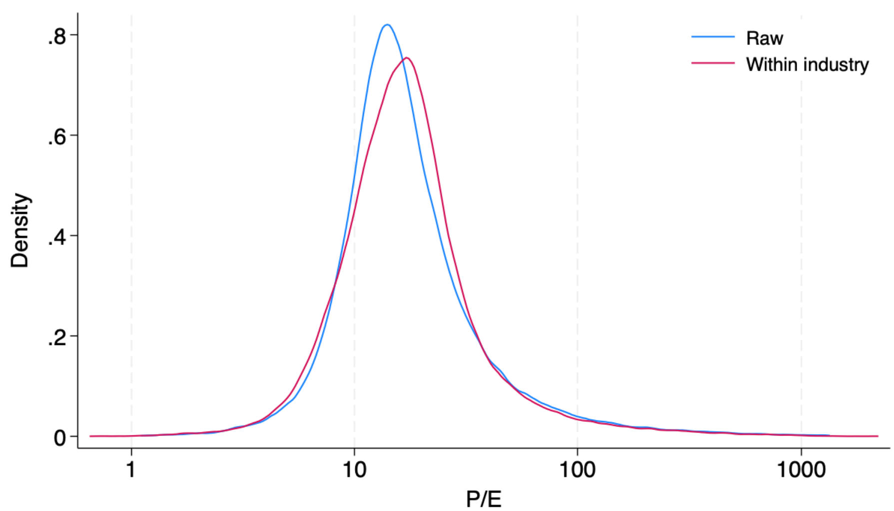
A Empirics

Figure A1: Distribution of P/E, removing year and fiscal month FE



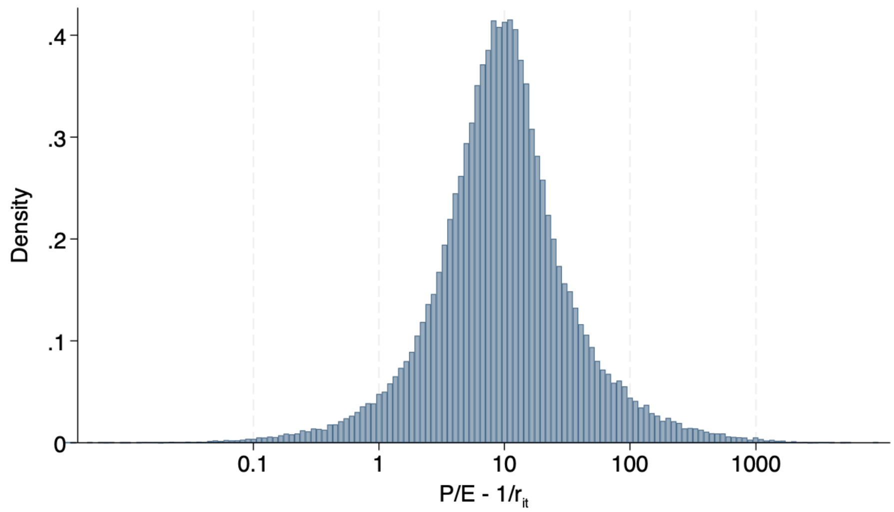
Source: CRSP and I/B/E/S.

Figure A2: Distribution of P/E, removing industry FE



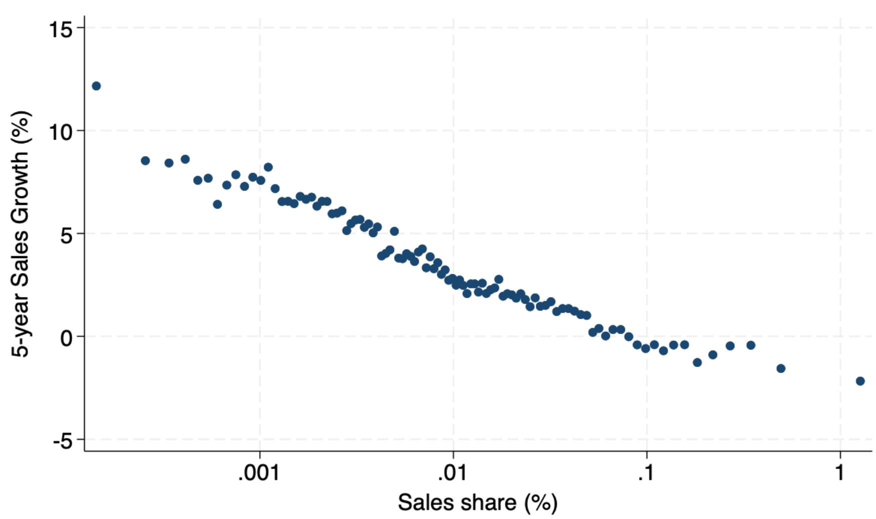
Source: CRSP and I/B/E/S.

Figure A3: Distribution of P/E, removing firm-specific risk



Source: CRSP and I/B/E/S. r_{it} from David et al. (2022)

Figure A4: Sales growth vs sales



Source: Compustat.

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