The Allocation of Talent
and U.S. Economic Growth

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Abstract

In 1960, 94 percent of doctors and lawyers were white men. By 2010, the fraction was just 62 percent. Similar changes in other highly-skilled occupations have occurred throughout the U.S. economy during the last fifty years. Given that the innate talent for these professions is not likely to have changed differently across groups, the change in the occupational distribution since 1960 suggests that a substantial pool of innately talented blacks and women in 1960 were not pursuing their comparative advantage. We examine the effect on aggregate productivity of the convergence in the occupational distribution between 1960 and 2010 through the prism of a Roy model. Across our various specifications, between one-fifth and two-fifths of growth in aggregate market output per person can be explained by the improved allocation of talent.

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1 Introduction

Over the last 50 years, there has been a remarkable convergence in the occupational distribution between white men, women, and blacks. For example, 94 percent of doctors and lawyers in 1960 were white men. By 2010, the fraction was just over 60 percent. Similar changes occurred throughout the economy during the last fifty years, particularly among highly-skilled occupations. A large literature attempts to explain these facts.\(^1\) Yet no formal study has assessed the effect of these changes on aggregate economic performance. Since the innate talent for a profession among members of group is unlikely to change over time, the change in the occupational distribution since 1960 suggests that a substantial pool of innately talented blacks and women in 1960 were not pursuing their comparative advantage. The resulting (mis)allocation of talent could potentially have important aggregate consequences.

This paper measures the aggregate effects of the changing allocation of talent from 1960 to 2010. To do so, we examine the differences in labor market outcomes between race and gender groups through the prism of a Roy (1951) model of occupational choice. Within the model, every person is born with a range of talents or preferences across all possible occupations. In an efficient allocation, each individual chooses the occupation where she obtains the highest return given her talents and preferences. We introduce three forces that will cause individuals to choose occupations where they do not have a comparative advantage. First, we allow for discrimination in the labor market. Consider the world that Supreme Court Justice Sandra Day O’Connor faced when she graduated from Stanford Law School in 1952. Despite being ranked third in her class, the only private sector job she could get after graduating was as a legal secretary (Biskupic, 2006). We model labor market discrimination as an occupation-specific wedge between wages and marginal products. This “tax” is a proxy for many common formulations of taste-based and statistical discrimination in the literature.\(^2\)

Second, the misallocation of talent can also be due to barriers to human capital investment. We model these barriers as increased monetary costs associated with accu-

\(^1\)See, for example, Blau (1998), Blau, Brummund and Liu (2013b), Goldin (1990), Goldin and Katz (2012), Smith and Welch (1989) and Pan (2015). Detailed surveys of this literature can be found in Altonji and Blank (1999), Bertrand (2011), and Blau, Ferber and Winkler (2013a).

\(^2\)See, for example, Becker (1957a), Phelps (1972) and Arrow (1973). A summary of such theories can be found in Altonji and Blank (1999).
mulating occupation-specific human capital. These costs are a proxy for many different race- and gender-specific factors. Examples include parental and teacher discrimination in favor of boys in the development of certain skills, historical restrictions on the admission of women to colleges or training programs, differences in school quality between black and white neighborhoods, differences in parental wealth and schooling levels across groups which alter the cost of investing in their children’s human capital.\(^3\)

Finally, we allow for differences in preferences or social norms to drive occupational choice differences across groups. For example, there might have been strong commonly held social norms against women and blacks in high-skilled occupations in the 1960s. The potential for preference or social norm differences across groups has been highlighted in the work of, among others, Johnson and Stafford (1998), Altonji and Blank (1999), and Bertrand (2011). We treat the home sector as additional occupation. As a result, we also allow for differences across groups in the extent to which they want to work in the home sector. This factor can capture changes in social norms related to women working at home. However, we can interpret the change in the preference for the home sector over time broadly so that it also includes changes in the preference for children or the ability to control the timing of fertility.\(^4\)

To measure these three forces from the data, we make a key assumption that the distribution of innate talent of blacks and women — relative to white men — is constant over time. With this assumption, we back out the change in labor market frictions, human capital frictions, and occupational preferences from synthetic panel data on

\(^3\)Here is an incomplete list of the enormous literature on these forces. Karabel (2005) documents how Harvard, Princeton, and Yale systematically discriminated against blacks, women, and Jews in admissions until the late 1960s. Card and Krueger (1992) document that public schools for blacks in the U.S. South in the 1950s were underfunded relative to schools for white children. See Chay, Guryan and Mazumder (2009) for evidence on the importance of improved access to health care for blacks. Goldin and Katz (2002), Bailey (2006), and Bailey, Hershbein and Milleri (2012) document that innovations related to contraception had important consequences for female labor market outcomes and educational attainment. Neal and Johnson (1996) document differences in AFQT scores across race and how controlling for AFQT explains a portion of Black-White gaps. Akcigit, Grigsby and Nicholas (2017) highlight the potential importance of parental liquidity constraints affecting investments into their children’s education. Therefore, educational attainment differences across groups can be determined by differences in parental background and parental wealth.

\(^4\)The literature on changes in female labor supply due to changes in productivity, preferences, and social norms is extensive. See Fernández, Fogli and Olivetti (2004) and Fernández (2013) on the role of cultural forces, Greenwood, Seshadri and Yorukoglu (2005) on the role of home durables, and Goldin and Katz (2002) on the role of birth control. Surveys on this literature can be found in Costa (2000) and Blau, Ferber and Winkler (2013a). There is also a literature on the changes in labor force participation rates for black men. Neal and Rick (2014), for example, highlights the importance of incarceration rates in explaining changes in labor market outcomes for black versus white men.
the occupational distribution and wages of women and blacks relative to white men from 1960 to 2010. Specifically, we infer that group-specific occupational preferences, labor market frictions, and human capital frictions must have declined from 1960 to 2010 to jointly explain the observed convergence in the occupational distribution and wages between blacks and women relative to white men. When we filter these facts through the lens of our general equilibrium model of occupational choice, we find that that changes in these frictions account for roughly two-fifths of growth in U.S. market GDP per person between 1960 and 2010. They also account for most of the rise in labor force participation over the last five decades.

We also use the structure of the model to decompose the contribution of each force. In our base specification, we assume all individuals draw a vector of occupation-specific productivities. With this assumption, wage differences across groups within an occupation discipline the role of common group-specific preferences in explaining cross-group differences in occupational choice. If women did not like being lawyers in 1960, the model implies that women should have been paid more to compensate for this disamenity. Second, we use the life cycle structure of the model to distinguish between barriers to human capital attainment and labor market discrimination. In our setup, human capital barriers affect an individual’s choice of human capital prior to entering the labor market. The effect of these barriers remains with a cohort throughout their life cycle. In contrast, labor market discrimination affects all cohorts within a given time period. Under these assumptions, we can use the changing differential life-cycle patterns of wages between groups to distinguish changing occupation-specific human capital barriers (which are akin to “cohort” effects) from changing occupation-specific labor market discrimination (which are akin to “time” effects).

We find that declining obstacles to accumulating human capital were much more important than declining labor market discrimination. Declining barriers to human capital attainment explain 36 percent of growth in U.S. GDP per person between 1960 and 2010, while declining labor market discrimination explains 8 percent of growth. Meanwhile, under our baseline model, changing common group specific occupational preferences explain little of U.S. growth during this time period.

In our base specification, individuals draw a vector of occupation-specific productivities. We document that many of our main empirical findings are both qualitatively
and quantitatively similar if we instead allow workers to draw a vector of occupation-specific preferences or a combination of occupation-specific productivities and preferences. Regardless of whether individuals draw occupation-specific preferences or productivities, we show that differences across groups in occupational propensities rather than in wage gaps is a more reliable way to infer occupational barriers. Additionally, even if individuals only sort on preferences instead of productivities, one-fifth of growth in market GDP per person over the last five decades can be traced to declining occupational barriers. A key finding is that women and black men are moving into high skilled occupations over time. In doing so, they are accumulating more human capital that raises aggregate productivity. The human capital effects occur regardless of whether the women and black men are moving into high skilled occupations because of declining frictions in those occupations or because of increasing preferences for those occupations.

Our paper adds to the large literature explaining differences in occupational sorting and wage gaps between race and gender groups in two important ways. First, we extend a Roy model of selection to explain differences in both occupational outcomes and wages across different race-sex groups. The model nests many of the stories highlighted in the literature to explain differential sorting patterns like labor market discrimination, barriers to human capital accumulation, and preference differences. Second, we use this model to assess how the changing occupational choice of women and blacks have contributed to U.S. economic growth over the last fifty years.

The rest of the paper proceeds as follows. Section 2 presents the model. Section 3 discusses data and inference for our baseline model in which individuals differ in occupational productivities. Section 4 presents the main results for this setting. Section 5 explores the robustness when individuals sort based on preferences or a combination of preferences and productivities. Section 6 discusses a variety of additional robustness exercises. Section 7 concludes.

2 Model

The economy consists of a continuum of workers, each in one of $M$ discrete sectors, one of which is the home sector. Workers are indexed by occupation $i$, group $g$ (such as
race and gender), and cohort $c$. A worker possesses heterogeneous abilities ($\epsilon_i$) or preferences ($\mu_i$) over occupations $i$. Some people are simply better teachers while others derive more utility from working as a teacher. The basic allocation to be determined in this economy is how to match workers with occupations.

### 2.1 Workers

In a standard Roy (1951) model, workers are endowed with idiosyncratic talent $\epsilon$ in each occupation (one of which is home production). We add two additional forces to the standard Roy (1951) model. First, we assume workers are heterogeneous in either their talent or their idiosyncratic preferences over occupations, but not both; heterogeneity on both dimensions hinders tractability. Second, we allow for forces that distort the allocation of workers across occupations. We have in mind forces such as discrimination in the labor market, barriers to human capital accumulation, and group-specific social norms.

Individuals invest in their human capital and choose an occupation in an initial "pre-period", after which they work in the chosen market occupation or home sector for three working life cycle periods ("young", "middle" and "old"). We assume that human capital investments and the choice of occupation are fixed after the pre-period.

Lifetime utility of a worker from group $g$ and cohort $c$ who chooses occupation $i$ is a function of lifetime consumption, time spent on human capital accumulation, and the preferences associated with choosing the occupation:

$$\log U = \beta \left[ \sum_{t=c}^{c+2} \log C(c, t) \right] + \log [1 - s(c)] + \log z_{ig}(c) + \log \mu$$

Here $C(c, t)$ is consumption of cohort $c$ in year $t$, $s$ denotes time allocated to human capital acquisition in the pre-period, $z_{ig}$ is the common utility benefit of all members of group $g$ from working in occupation $i$, $\mu$ is the idiosyncratic utility benefit of the individual from the occupation, and $\beta$ parameterizes the trade-off between lifetime consumption and time spent accumulating human capital.$^5$ We normalize the time endowment in the pre-period to 1 so $1 - s$ is leisure time in the pre-period. Forces

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$^5$We define the cohort index $c$ as the time when the cohort is young so time $t = c$ is the first period of cohort $c$. We omit subscripts on other individual-specific variables to keep the notation clean. However, $z_{ig}$ does have subscripts to emphasize that it varies across groups and occupations.
such as changes in social norms for women working in the market sector or changing common preferences in fertility and marriage patterns can be thought of as changes in $z$ of the home sector of women (but not of men). The idiosyncratic preference of a specific women in a given occupation is represented by $\mu$. To be clear, $C$, $s$, and $\mu$ are all individual specific and depend on occupation and group as well.

Individuals acquire human capital in the initial period, and this human capital remains fixed over their lifetime. Individuals use time $s$ and goods $e$ to produce $h$:

$$h_{ig}(c, t) = \bar{h}_{ig} \gamma(t - c) s_i(c)^{\phi_i} e_{ig}(c)^{\eta}. $$

$\bar{h}_{ig}$ captures permanent differences by group-occupation pairs in human capital endowments and $\gamma$ parameterizes the return to experience. We assume $\gamma$ is only a function of age $= t - c$ and $\bar{h}_{ig}$ is fixed for a given group-occupation. $\bar{h}_{ig}$ could include early investments in nutrition, health, cognitive development resulting from differing socioeconomic backgrounds, or simply natural differences in talent common to a group in a given occupation. $\phi_i$ is the occupation specific return to time investments in human capital accumulation while $\eta$ is the elasticity of human capital with respect to human capital expenditures.

Consumption in each period is net income minus a portion of expenditures that are spent on education:

$$C(c, t) = \left[1 - \tau_{wg}(t)\right] w_i(t) e h_{ig}(c, t) - e_{ig}(c, t) \left[1 + \tau_{ig}^h(c)\right]. $$

(2)

Net income is the product of $1 - \tau_{wg}$ and the total efficiency units of labor, which is the product of the price per efficiency unit of skill $w_i$, the idiosyncratic talent in the worker’s chosen occupation $e$ and human capital $h$. Individuals borrow $e(c)(1 + \tau_{ig}^h(c))$ in the first period to purchase $e(c)$ units of human capital, a loan they repay over their lifetime subject to the lifetime budget constraint $e(c) = \sum_{t=c}^{c+2} e(c, t)$.

Labor market discrimination $\tau_{wg}$ works as a “tax” on individual earnings. Given our assumption that the firm owner discriminates against all workers of a given group, $\tau_{wg}$ affects all the cohorts of group $g$ within occupation $i$ equally at a given point in time. Barriers to human capital attainment $\tau_{ig}^h$ affect consumption directly by increasing the cost of $e$ for a given group-occupation pair in (2) as well as indirectly by lowering ac-
quired human capital $\epsilon$. We interpret $\tau_{ig}^h$ broadly to incorporate even early differences in childhood environments across groups, as long as these differences affect accumulation of human capital. That is, $\tau_{ig}^h$ reflects more than just discrimination in access to quality schooling. Because the human capital decision is only made once and fixed thereafter, $\tau_{ig}^h$ for a given occupation varies across cohorts and groups, but is fixed for a given cohort-group over time.

Given an occupational choice, the occupational wage $w_i$, and idiosyncratic ability $\epsilon$ in the occupation, the individual chooses consumption in each period and $e$ and $s$ in the initial pre-period to maximize lifetime utility given by (1) subject to the constraints given by (2) and $e(c) = \sum_{t=0}^{\infty} e(c,t)$. Individuals will choose the time path of $e(c,t)$ such that expected consumption is constant and equals one third of expected lifetime income. Lifetime income depends on $\tau_{ig}^h$ in the first period (when the individual is young) and the expected values of $w_i$, $\tau_{ig}^w$, and $\gamma$ in middle and old age. For simplicity, we assume that individuals anticipate that the return to experience varies by age but that the labor tax $\tau_{ig}^w$ and returns to market skill $w_i$ they observe when young will remain constant over time. Because individuals expect the same conditions in future periods as in the first period (except for the accumulation of experience), expected lifetime income is proportional to income in the first period.

The amount of time and goods an individual spends on human capital are then:

$$s_i^* = \frac{1}{1 + \frac{1-\eta}{\beta \phi_i}}$$

$$e_{ig}^* = \left( \frac{\eta(1 - \tau_{ig}^w)w_i \tilde{\gamma} h_{ig} s_i^\phi \epsilon}{1 + \tau_{ig}^h} \right)^{\frac{1}{1-\eta}}$$

where $\tilde{\gamma} = 1 + \gamma(1) + \gamma(2)$ is the sum of the experience terms over the life-cycle with $\gamma(0)$ set to 1. Time spent accumulating human capital is increasing in $\phi_i$. Individuals in high $\phi_i$ occupations acquire more schooling and have higher wages as compensation for time spent on schooling. Forces such as $w_i$, $h_{ig}$, $\tau_{ig}^h$, and $\tau_{ig}^w$ do not affect $s$ because they have the same effect on the wage gains from schooling and on the opportunity cost of time. These forces do change the return to investing goods in human capital (relative to the cost) with an elasticity that is increasing in $\eta$. These expressions hint at why we use both time and goods in the production of human capital. Goods are needed so that
distortions to human capital accumulation matter. As we show below, time is needed to explain average wage differences across occupations.

After substituting the expression for human capital into the utility function, indirect expected utility for an individual from group \( g \) working in occupation \( i \) is

\[
U_{ig}^* = \mu_i \left[ \gamma \bar{w}_{ig} \epsilon_i \right]^{\frac{3\beta}{1-\eta}}
\]

where

\[
\bar{w}_{ig} \equiv w_i s_i \phi_i (1 - s_i)^{\frac{1-n}{3\beta}} \cdot \frac{\bar{h}_{ig}}{\bar{r}_{ig}} \cdot \bar{z}_{ig},
\]

\[
\tau_{ig} \equiv \frac{(1 + \tau_{ig}^h)^{\eta}}{1 - \tau_{ig}^w}
\]

and

\[
\tilde{z}_{ig} \equiv z_{ig}^{\frac{1-n}{3\beta}}.
\]

The effect of labor market discrimination and human capital barriers is summarized by the “composite” \( \tau_{ig} \). More human capital barriers or labor market discrimination increase \( \tau_{ig} \), which lowers indirect utility for an individual from group \( g \) when choosing occupation \( i \). Group-specific disutility from working in occupation \( i \) is represented as a low value of \( \tilde{z}_{ig} \). We represent group-specific preferences by \( \tilde{z} \) instead of \( z \) to make the units of group preferences comparable to units of \( \tau \). Higher innate talent \( \epsilon \) or innate preferences \( \mu \) also increases the rewards for choosing an occupation. So the effect of idiosyncratic talent on occupational choice is isomorphic to the effect of idiosyncratic utility.

Finally, turning to the distribution of the idiosyncratic talent \( \epsilon \) and preferences \( \mu \), we borrow from McFadden (1974) and Eaton and Kortum (2002). Each person either gets a skill draw \( \epsilon_i \) or a preference draw \( \mu_i \) in each of the \( M \) occupations. To be clear, if a worker gets a skill draw, we assume that \( \mu = 1 \) for the worker. And if the worker gets a utility draw, then \( \epsilon \) is the same for all such individuals. If the person gets a skill draw, talent in each occupation is drawn from a multivariate Fréchet distribution:

\[
F_g(\epsilon_1, \ldots, \epsilon_M) = \exp \left[ -\sum_{i=1}^{M} \epsilon_i^{-\beta} \right].
\]
The parameter $\theta$ governs the dispersion of skills, with a higher value of $\theta$ corresponding to smaller dispersion. We normalize the mean parameter of the skill distribution to one in all occupations for all groups, but this mean parameter is isomorphic to $\bar{h}_{ig}$.

If the individual instead gets a preference draw, these preferences are also drawn from a multivariate Fréchet distribution, where the shape parameter for the Fréchet distribution of preferences is equal to $\frac{\theta(1-\eta)}{3\beta}$. This assumption makes the elasticity of labor supply to an occupation of individuals with heterogeneous preferences the same as that of workers with ability heterogeneity. We assume the ability of workers that sort on preferences is given by $\epsilon_i = \Gamma^{1-\eta}$ where $\Gamma \equiv \Gamma \left(1 - \frac{1}{\theta(1-\eta)}\right)$ is the Gamma function. This assumption makes average ability the same for the two groups of workers.

### 2.2 Occupational choice

Given the above assumptions, the occupational choice problem thus reduces to picking the occupation that delivers the highest value of $U^*_ig$.

Because heterogeneity is drawn from an extreme value distribution, the highest utility can also be characterized by an extreme value distribution, a result reminiscent of McFadden (1974). The overall occupational share can then be obtained by aggregating the optimal choice across people.\(^6\)

\textbf{Proposition 1 (Occupational Choice):} Let $p_{ig}(c)$ denote the fraction of people from cohort $c$ and group $g$ who choose occupation $i$, a choice made when they are young. Aggregating across people, the solution to the individual’s choice problem leads to

$$p_{ig}(c) = \frac{\tilde{w}_{ig}(c)^\theta}{\sum_{s=1}^{M} \tilde{w}_{sg}(c)^\theta} \quad (4)$$

where $\tilde{w}_{ig}(c) \equiv w_i(c)s_i(c)\phi_i(c) [1 - s_i(c)]^{1-\eta} \cdot \frac{h_{ig} z_{ig}(c)}{\bar{w}_{ig}(c)}$.

Occupational sorting depends on $\tilde{w}_{ig}$, which is the overall reward that someone from group $g$ with the mean talent obtains by working in occupation $i$, relative to the power mean of $\bar{w}$ for the group over all occupations. The occupational distribution is driven by relative returns and not absolute returns: forces that change $\bar{w}$ for all occupations have

\(^6\)Proofs of the propositions are given in the Appendix which can be found at http://www.stanford.edu/~chadj/HHJKAppendix.pdf.
no effect on the occupational distribution. Occupations where the wage per efficiency unit \( w_i \) is high will attract more workers of all groups.

In contrast, differences between groups in occupational choice are driven by differences in \( \tilde{z}_{ig} \), \( h_{ig} \), \( \tau^w \), and \( \tau^h \). The fraction of members of group \( g \) that choose occupation \( i \) is low when a group commonly dislikes like the occupation (\( \tilde{z}_{ig} \) is low), have low ability in the occupation (\( h_{ig} \) is low), employers discriminate against the group in the occupation (\( \tau^w_{ig} \) is high), or when the group faces a barrier in accumulating human capital associated with that occupation (\( \tau^h_{ig} \) is high).

We view home production as simply another occupation so the share of a group in the home sector is also given by equation (4). The labor force participation rate therefore depends on the return in the home sector relative to the market sectors. For example, the decline in the labor force participation rate of white men since the 1960s can be driven by higher returns in the home sector (such as better video games), a decline in labor market opportunities (such as the decline of blue-collar jobs) or changing preferences for the market sector relative to the home sector. Or the increase in female labor force participation rates from 1960 to 2010 can be due to less labor market and human capital discrimination in market occupations.

### 2.3 Worker Quality

For individuals with heterogeneous abilities, sorting affects the average quality of workers in an occupation for each group. For individuals with heterogeneous preferences, sorting has no effect on the average quality of workers in an occupation. While sorting on productivities or preferences have a similar effect on the number of individuals from a given group working in a given occupation, they have different effects on occupational wages. Average worker quality in an occupation is therefore a weighted average of the quality of workers who sort on ability and those who sort on preferences:

**Proposition 2** (Average Quality of Workers): For a given cohort \( c \) of group \( g \) at time \( t \), the geometric average of worker quality in each occupation, including both human capital
and talent, is

\[
e^{E \log [h_{ig}(c,t) \epsilon_{ig}(c)]} = \Gamma s_i(c) \phi_i(t) \gamma(t-c) \left( \frac{\eta s_i(c) \phi_i(c) \gamma \bar{h}_{ig} w_i(c) [1 - \tau_{ig}^w(c)]}{1 + \tau_{ig}^h(c)} \right)^{1-\eta} \left( \frac{1}{p_{ig}(c)} \right)^{\frac{1-\epsilon}{\eta(1-\eta)}}
\]

(5)

Here \( \delta \) is the share of the population with idiosyncratic preferences (so \( 1 - \delta \) is the share of workers with idiosyncratic ability) and \( \bar{\Gamma} \) is a constant.\(^7\) By varying \( \delta \), we can explore the robustness of our results to sorting that occurs completely on talent (\( \delta = 0 \)), sorting that occurs completely on preferences (\( \delta = 1 \)), or sorting that occurs on both margins. When all individuals possess heterogeneous abilities (\( \delta = 0 \)), average quality is inversely related to the share of the group working in the occupation \( p_{ig}(c) \). This captures the selection effect. For example, the model predicts that if the labor market discriminated against female lawyers in 1960, only the most talented female lawyers would have chosen to work in this occupation. And if the barriers faced by female lawyers declined after 1960, less talented female lawyers would move into the legal profession and thus lower the average quality of female lawyers. Conversely, in 1960, the average quality of white male lawyers would have been lower in the presence of labor market discrimination against women and blacks. At the other extreme, when \( \delta = 1 \) (all workers sort on preferences), this selection effect is absent.

2.4 Occupational Wages

Next, we compute the average wage for a given group working in a given occupation — the model counterpart to what we observe in the data.

**Proposition 3** (Occupational Wages): Let \( \text{wage}_{ig}(c,t) \) denote the geometric average of earnings in occupation \( i \) by cohort \( c \) at date \( t \) of group \( g \). Its value satisfies

\[
\text{wage}_{ig}(c,t) \equiv (1 - \tau_{ig}^w(t)) w_i(t) e^{E \log [h_{ig}(c,t) \epsilon_{ig}]}
\]

\[
= \bar{\Gamma} \bar{\eta} \left[ p_{ig}(c) \delta m_g(c) \right]^{\frac{1}{\eta(1-\eta)}} \bar{z}_{ig}(c) - \frac{1}{1 - s_i(c)} [1 - s_i(c)]^{-\frac{1}{\eta}} \frac{1 - \tau_{ig}^w(t)}{1 - \tau_{ig}^h(c)} \frac{w_i(t) \gamma(t-c)}{\gamma_i(c) \phi_i(t)}
\]

where \( m_g(c) \equiv \sum_{i=1}^{M} \tilde{w}_{ig}(c) \theta \) and \( \bar{\eta} \equiv \eta^{\eta/(1-\eta)} \).

\(^7\) \( \bar{\Gamma} \) is defined in equation (A7) in the Appendix.
For individuals in the young cohort, \( t = c \) which implies
\[
\frac{s_i(c)^{\phi_i(t)}}{s_i(c)^{\phi_i(c)}} = 1 \quad \text{and} \quad \frac{(1-t_{ig}^w(t))w_i(t)}{(1-t_{ig}^s(c))w_i(c)} = 1.
\]
When all individuals sort on ability (\( \delta = 0 \)), average earnings for a given group among the young differs across occupations only because of differences in \( s_i \) and \( \tilde{z}_{ig} \). Occupations in which schooling is especially productive (a high \( \phi_i \) and therefore a high \( s_i \)) will have higher average earnings. Similarly, occupations where individuals have a strong common dis-utility from being in the profession (\( \tilde{z}_{ig} \) is small) have higher wages as compensation for the lower utility. These are the only two forces that generate differences in wages across occupations for the young when individuals sort completely on talent (\( \delta = 0 \)). Average earnings are no higher in occupations where a group faces less discrimination in the labor market, lower frictions in human capital attainment, a higher wage per efficiency unit, or where the group has more talent in the sector. The reason is that each of these factors leads lower quality workers to enter those jobs. This composition effect exactly offsets the direct effect on earnings when the distribution of talent is Fréchet.

The composition effect would not be present if selection was driven by forces other than occupational ability. When all workers select based on idiosyncratic preferences (\( \delta = 1 \)), then selection affects the average utility of workers in an occupation, but has no effect on average ability. In this case, there is no quality offset due to selection and the average wage in an occupation varies with \( p_{ig} \). Therefore, the average wage and the occupational share will both be higher in occupations where a group faces less discrimination or where the wage per efficiency unit is higher.

The general point however is that the wage gap is not a robust measure of the frictions faced by a group in a given occupation. The elasticity of the wage gap to the occupational frictions depends on the share of individuals who sort on preferences instead of ability. When individuals sort on ability (\( \delta = 0 \)), the wage gap is uncorrelated with the these frictions because of the offsetting effect of selection.

Equation (6) for the average wage also identifies the forces behind wage changes over a cohort’s life-cycle. For a given cohort-group in an occupation, \( s_i, \tilde{z}_{ig} \), and \( p_{ig} \) are fixed. Therefore, the average wage increases over time when the price of skills in the occupation \( w_i \) increases, labor market discrimination \( \tau^w \) falls, return to experience is positive, or the return to schooling increases.

Comparing wage gaps in an occupation across groups, the returns to schooling,
experience, and returns to skill have the same effect on all groups (of a given cohort in the occupation). For the young cohort, the wage gap differs across occupations only if relative $\tilde{z}_{ig}$ differs across occupations. And since $\tilde{z}_{ig}$ is constant for a given cohort, differences in the growth rate of wages between groups (say between men and women) can only be due to differences in the change in $\tau^w$ between the groups. This is the case regardless of whether individuals sort by ability or by preferences.

### 2.5 Relative Propensities

Putting together the equations for the occupational shares and wages in each occupation and assuming the experience profiles are the same across groups, we get the relative propensity of a group to work in an occupation. These equations provide us a way to explain differences in occupational choice through the lens of our model that can be mapped directly to observable data moments.

**Proposition 4** (Relative Propensities): *The fraction of a group working in an occupation — relative to white men — is given by*

$$
\frac{p_{ig}(c)}{p_{i,wm}(c)} = \left( \frac{\tau_{ig}(c)}{\tau_{i,wm}(c)} \right)^{\frac{\theta}{\theta - \delta}} \left( \frac{\tilde{h}_{ig}}{\tilde{h}_{i,wm}} \right)^{\frac{\theta}{\theta - \delta}} \left( \frac{\text{wage}_{ig}(c,c)}{\text{wage}_{i,wm}(c,c)} \right)^{-\frac{\theta(1-\eta)}{\theta - \delta}}
$$

(7)

*where the subscript “wm” denotes white men.*

The propensity of a group to work in an occupation (relative to white men) depends on three terms: the relative composite occupational frictions, relative talent in the sector, and the average wage gap between the groups in the occupation. From Proposition 3, the wage gap itself is a function of the distortions faced by the group, the talent of the group, and the price of skills in all occupations. With data on occupational shares and wages, we can measure a composite term that measures the combined effect of labor market discrimination, barriers to human capital attainment, and talent in the sector. The preference parameters $\tilde{z}_{ig}$ do not enter this equation once we have controlled for the wage gap; instead, they influence the wage gaps themselves.
2.6 Relative Labor Force Participation

The labor force participation rate of a group relative to white men is also given by equation (7). We normalize $\tilde{z} = 1$, $\tau^w = 0$ and $\tau^h = 0$ for the home sector. With these normalizations, the gap in labor force participation rate relative to white men is given by:

**Proposition 5** (Relative Labor Force Participation): Let $LFP_g \equiv 1 - p_{\text{home},g}$ denote the share of group $g$ in the market occupations. The share of group $g$ in the home sector relative to white men is then

\[
\frac{1 - LFP_g(c)}{1 - LFP_{wm}(c)} = \frac{m_{wm}(c)}{m_g(c)} = \left( \frac{\text{wage}_{ig}(c, c)}{\text{wage}_{i,wm}(c, c)} \right)^{-\theta(1-\eta)} \left( \frac{\tilde{z}_{ig}(c)}{\tilde{z}_{i,wm}(c)} \right)^{-\theta} \left( \frac{p_{ig}(c)}{p_{i,wm}(c)} \right)^{\delta} \forall i \in \text{market}
\]

where $\frac{m_{wm}(c)}{m_g(c)} = \frac{\sum_{i=1}^{M} \tilde{w}_{i,wm}(c)^\theta}{\sum_{i=1}^{M} \tilde{w}_{ig}(c)^\theta}$.

Since the return to the home sector sector is the same for all groups (this is an implication of the normalization that the home sector is undistorted), $\frac{m_{wm}(c)}{m_g(c)}$ is the return to market work of white men relative to group $g$. For example, if women are discriminated against in the labor market or in accumulating human capital for the market sector, this will drive down female labor force participation rates. Or if social norms discourage women from the market sector (low $\tilde{z}$ in market sectors), this will also lower female labor force participation rates.

The second equation in (8) says that the relative return to market work is given by a power function of the gap in market wages in any market sector, the relative occupational preference term in that sector and the relative occupational propensity in the sector with an elasticity that depends on the share of people that sort on preferences $\delta$. We will use this insight to back out $\tilde{z}$ in the market sectors from data on labor force participation of the group and wage gaps, both relative to white men.
2.7 Firms and Determinants of Labor Market and Human Capital Frictions

A representative firm produces final output $Y$ from workers in $M$ occupations:

$$Y = \left[ \sum_{i=1}^{M} \left( A_i \cdot \sum_{g} H_{ig} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}}$$

(9)

where $H_{ig}$ denotes the total efficiency units of labor provided by group $g$ in occupation $i$ and $A_i$ is the exogenously-given productivity of occupation $i$. The parameter $\sigma$ represents the elasticity of substitution across occupations in aggregate production.

Following Becker (1957a), we assume the owner of the firm in the final goods sector discriminates against workers of certain groups. We model the “taste” for discrimination as lower utility of the owner when she employs workers from groups she dislikes. Her utility is given by

$$U_{owner} = Y - \sum_{i} \sum_{g} (1 - \tau_{wg}) w_i H_{ig} - \sum_{i} \sum_{g} d_{ig} H_{ig}$$

(10)

The first term denotes profit and the second term captures the extent to which owners are prejudiced: $d_{ig}$ is the utility loss associated with employing workers from group $g$ in occupation $i$. Because all employers are assumed to have these racist and sexist preferences, perfect competition implies that $\tau_{wg} = d_{ig} / w_i$. Intuitively, when the owner hires a worker from a group she dislikes, she needs to be compensated for her utility loss via a lower wage for these workers. In equilibrium the utility loss is exactly offset by the lower wage. Thus the frictions are ultimately pinned down by the discriminatory tastes of (homogeneous) owners.\(^8\)

A second firm (a “school”) sells educational goods $e$ to workers who use it as an input in their human capital. We assume the school’s owner dislikes providing $e$ to

\(^8\)What is important is not that all firms discriminate but that the marginal firm discriminates (Becker, 1957b). We abstract from firm heterogeneity and instead assume all firms within an occupation discriminate. This simplifies the analysis but still allows us to match key features of the data.
certain groups. The utility of the school’s owner is

\[
U_{\text{school}} = \sum_i \sum_g \left( R_{ig} - \left( 1 - \tau_{ig}^h \right) \right) \cdot e_{ig} - \sum_i \sum_g d_{ig}^h e_{ig} \tag{11}
\]

where \( e_{ig} \) denotes educational resources provided to workers from group \( g \) in market sector \( i \), \( R_{ig} \) denotes the price of \( e_{ig} \), and \( d_{ig}^h \) represents the owner’s distaste from providing educational resources to workers from group \( g \) in sector \( i \). We think of this as a shorthand for complex forces such as discrimination against blacks or women in admission to universities, or differential allocation of resources to public schools attended by black vs. white children, or differential parental investments made toward building up math and science skills in boys relative to girls. Groups that are discriminated against in the provision of human capital pay a higher price for \( e \), and the higher price compensates the school owner for her disutility. Perfect competition ensures that \( R_{ig} = 1 \), and that \( \tau_{ig}^h = d_{ig}^h \).

### 2.8 Equilibrium

A competitive equilibrium in this economy consists of a sequence of individual choices \( \{C, e, s\} \), occupational choices in the pre-periods, total efficiency units of labor of each group in each occupation \( H_{ig} \), final output \( Y \), and an efficiency wage \( w_i \) in each occupation such that

1. Given an occupational choice, the occupational wage \( w_i \), and idiosyncratic ability \( \epsilon \) in that occupation, each individual chooses \( C, e, s \) to maximize expected lifetime utility given by (1) subject to the constraints given by (2) and \( e(c) = \sum_{t=c}^{c+2} e(c, t) \).

2. Each individual chooses the occupation that maximizes expected lifetime utility:

\[
i^* = \arg \max_i U(\tau_{ig}^w, \tau_{ig}^h, z_{ig}, w_i, \epsilon_i, \mu_i), \text{ taking as given } \{\tau_{ig}^w, \tau_{ig}^h, z_{ig}, \bar{h}_{ig}, \epsilon_i, \mu_i\}.
\]

3. A representative firm in the final good sector hires \( H_{ig} \) in each occupation to maximize profits net of utility cost of discrimination given by equation (10).

4. A representative firm in the education sector maximizes profit net of the utility cost of discrimination given by equation (11).
5. Perfect competition in the final goods and education sectors generates $\tau_{ig}^w = d_{ig}/w_i$ and $\tau_{ig}^h = d_{ig}^h$.

6. $w_i(t)$ clears each occupational labor market.

7. Total output is given by the production function in equation (9).

The equations characterizing the general equilibrium are given in the next result.

**Proposition 6** (Solving the General Equilibrium): The general equilibrium of the model is $H_{ig}^{\text{supply}}$, $H_i^{\text{demand}}$, $w_i$, and market output $Y$ at each point in time such that

1. $H_{ig}^{\text{supply}}(t)$ aggregates the individual choices:
   \[
   H_{ig}^{\text{supply}}(t) = \sum_c q_g(c)p_{ig}(c) \mathbb{E} \left[ h_{ig}(c)e_{ig}(c) \mid \text{Person chooses } i \right]
   \]
   where $q_g(c)$ denotes the number of workers of group $g$ and cohort $c$ and the average quality of workers is given in equation (5).

2. $H_i^{\text{demand}}(t)$ satisfies firm profit maximization:
   \[
   H_i^{\text{demand}}(t) = \left( \frac{A_i(t)^{\frac{\sigma-1}{\sigma}}}{w_i(t)} \right)^\sigma Y(t)
   \]

3. $w_i(t)$ clears each occupational labor market: $\sum_g H_{ig}^{\text{supply}}(t) = H_i^{\text{demand}}(t)$.

4. Total output is given by the production function in equation (9) and equals aggregate wages plus total revenues from $\tau^w$.

**2.9 Intuition**

To develop intuition, consider the following simplified version of the model. First, assume only two groups, men and women, and that men face no distortions. Second, assume occupations are perfect substitutes ($\sigma \to \infty$) so that $w_i = A_i$. With this assumption, the production technology parameter pins down the wage per unit of human capital in each occupation. In addition, labor market and human capital frictions affect aggregate output produced by women but have no effect on output produced by men.
Third, assume $\phi_i = 0$ (no schooling time), $\bar{z}_i = 1$, $\bar{h}_i = 1$, each cohort lives for one period, and that individuals only draw occupational talent ($\delta = 0$).

Aggregate output can then be expressed as the sum of aggregate output produced from male labor and female labor:

$$Y = q_m \cdot \left( \sum_{i=1}^M A_i^\theta \right)^{1/\theta} + \frac{q_w}{1 - \bar{\tau}_w} \cdot \left( \sum_{i=1}^M \left( A_i \left( 1 - \tau_i^w \right) \right)^\theta \right)^{1/\theta} \cdot 1 - \eta \cdot \ln (1 + \bar{\tau}_h) - \frac{\theta - 1}{2} \cdot \frac{1}{1 - \eta} \cdot \text{Var} \ln (1 + \tau_i^h).$$

(12)

where $q_w$ and $q_m$ denote the number of women and men, and $\bar{\tau}_w$ denotes the earnings-weighted average of the labor market friction facing women. The first term in (12) is aggregate output produced by men and is not affected by the occupational distortions facing women (this is driven by the assumption that occupations are perfect substitutes). The second term is aggregate output produced by female labor. So the effect of $\tau^w$ and $\tau^h$ on aggregate output shows up in the second term, and their effect on aggregate output is increasing in the number of people in the discriminated group $q_w$.

We illustrate how this setup can be used to gain intuition by focusing on $\tau^w$; the effects of $\tau^h$ can be analyzed in a similar fashion. Assuming $\tau^h = 0$ and that $\tau^w$ and $A$ are jointly log-normally distributed, aggregate output produced by women $Y_w$ (the second term in equation (12)) is given by

$$\ln Y_w = \ln q_w + \ln \left( \sum_{i=1}^M A_i^\theta \right)^{1/\theta} + \frac{\eta}{1 - \eta} \cdot \ln (1 - \bar{\tau}_w) - \frac{\theta - 1}{2} \cdot \frac{1}{1 - \eta} \cdot \text{Var} \ln (1 - \tau_i^w).$$

(13)

$\tau^w$ affects output via the last two terms in equation (13). The mean of $\tau^w$ changes the return to investment in human capital. This effect is captured by the third term in equation (13) and its magnitude depends on elasticity of output with respect to human capital $\eta$. The dispersion of $\tau^w$ across occupations affects aggregate output via a different channel. Here, dispersion of $\tau^w$ affects the allocation of female labor across occupations. A decline in the dispersion of $\tau^w$ improves the allocation, which increases...
aggregate output. This effect is captured by the fourth term in equation (13).

Finally, (13) suggests that the effect of unequal barriers on aggregate output is increasing in \( \theta \). While this is true for a given variance of “tax rates”, our inference about the magnitude of that variance from observed data also depends on \( \theta \). Using the equation for relative propensities, the variance in the labor distortion is given by:

\[
\text{Var} \ln(1 - \tau^w) = \frac{1}{\theta^2} \cdot \text{Var} \ln \frac{p_{ig}}{P_{i,wm}}
\]

This says that, conditional on data on occupational shares, the implied dispersion of \( \tau^w \) is decreasing in \( \theta \). Expressed as a function of data on occupational propensities, aggregate output from female labor is:

\[
\ln Y_w = \ln q_w + \ln \left( \sum_{i=1}^{M} A_i^\theta \right) \frac{\eta}{1-\eta} \cdot \ln (1 - \bar{\tau}^w) - \frac{1}{2} \cdot \frac{\theta - 1}{(1 - \eta)\theta^2} \cdot \text{Var} \ln \left( \frac{p_{ig}}{P_{i,wm}} \right)
\]

The elasticity of \( Y_w \) with respect to the variance in the observed propensities in the data is \( \frac{\eta}{2} \cdot \frac{\theta - 1}{(1 - \eta)\theta^2} \) while the elasticity with respect to the variance in \( \tau^w \) is \( \frac{\eta}{2} \cdot \frac{\theta - 1}{1 - \eta} \). Intuitively, a higher value of \( \theta \) implies that a given amount of misallocation has a larger effect on aggregate output. On the other hand, given the observed data on occupational shares, a higher \( \theta \) also implies a smaller amount of misallocation. For this reason, as we document later, the effect of changes in occupational shares on output growth will not be overly sensitive to the values we use for \( \theta \).

3 Inference with Selection Only on Ability (\( \delta = 0 \))

We now illustrate how we uncover the driving forces of the model from data on wages and occupational shares of different groups of workers. We begin by considering the case where all individuals only draw occupational talent (\( \delta = 0 \)). By examining this polar case, we provide the broad intuition of how observable data can be used to discipline the model. In Section 5, we discuss the inference exercise when selection occurs on both ability and preferences (\( \delta > 0 \)). In that section, we also provide an estimate of \( \delta \). The results with our estimated \( \delta \) are not far from our results with \( \delta = 0 \). As a result, we will refer to the \( \delta = 0 \) case as our benchmark specification.

\[^{11}\text{We maintain the assumption that } \tau^w \text{ is the only source of variation.}\]
The inference exercise — for any value of \( \delta \) — is based on two key assumptions. First, we assume the relative mean latent occupational talent of a group relative to white men \( \bar{h}_{ig}/\bar{h}_{i,wm} \) is constant over time. This is a key assumption and we cannot proceed without it. It implies that the change in the occupational distribution of women and blacks relative to white men since 1960 must be driven by changes in labor market or human capital frictions or by changes in common occupational preferences. Second, we assume that idiosyncratic occupational abilities or preferences are distributed iid Fréchet. This assumption is not as crucial, but it buys us enormous tractability because it leads to simple reduced form expressions for occupational shares and wages as a function of the occupational frictions. Relaxing this assumption is a valuable direction for future research, but would not change the fact that the \( \tau \)'s and \( \tilde{\varepsilon} \)'s must have changed since the 1960s to explain the observed changes in the distribution of occupations of white women and blacks relative to white men over the last fifty years.\(^{12}\)

This section proceeds in the following manner. First, we describe the data. Second, we explain what in the data allows us to distinguish between the composite of the labor and human capital frictions \( \tau \) and common group specific occupational preferences \( \tilde{\varepsilon} \). Third, we show what in the data allows us to separate the effect of labor market discrimination \( (\tau^w) \) from that of human capital barriers \( (\tau^h) \) in the composite \( \tau \). Fourth, we explain how we infer productivity growth in each occupation, the return to skill and preferences for market occupations relative to the home sector.

### 3.1 Data

We use data from the 1960, 1970, 1980, 1990, and 2000 decennial Censuses and the 2010–2012 American Community Surveys (ACS).\(^{13}\) We restrict the sample to white men, white women, black men and black women, which are the four groups we analyze. We also only include individuals between the ages of 25 and 54. This restriction focuses the analysis on individuals after they finish schooling and prior to retirement. Finally, we exclude individuals who report being unemployed (not working but searching for

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\(^{12}\)Lagakos and Waugh (2013) and Adão (2016) estimate selection models with arbitrary correlation but only with 2 or 3 sectors. We do not know how to do something similar for the nearly 70 occupations we have in our data.

\(^{13}\)When using the 2010–2012 ACS data, we pool all three years together for power and treat them as one cross section. Henceforth, we refer to the pooled 2010–2012 sample as the 2010 sample.
work) or on active military duty.\textsuperscript{14}

We create pseudo-panels by following synthetic cohorts from 1960 to 2010 over their life-cycle. We define three age periods within a cohort’s life cycle: young (age 25-34), middle (age 35-44) and old (age 45-54). For example, a synthetic cohort would be the young in 1960, the middle aged in 1970, and the old in 1980. We have information on 8 cohorts for the time periods we study. We observe information at all three life cycle points for 4 cohorts (the young in 1960, 1970, 1980 and 1990) and one or two life-cycle points for the remaining cohorts.

We define a person as either in the home sector or in the market sector based on their number of hours worked. We classify a person as being in the home sector if she is not currently employed or works less than ten hours per week. Individuals working more than thirty hours per week are classified as employed in one of 67 market occupations.\textsuperscript{15} Those who are employed but usually work between ten and thirty hours per week are classified as part-time workers. We split the sampling weight of part-time workers equally between the home sector and the reported market occupation.

We measure earnings as the sum of labor, business, and farm income in the previous year. For earnings we restrict the sample to individuals who worked at least 48 weeks during the prior year, who earned at least 1000 dollars (in 2007 dollars) in the previous year, and who reported working more than 30 hours per week. We convert all earnings data from the Census to constant dollars. Our measure of wage gaps across occupations and groups is the difference in the log of the geometric average of earnings.\textsuperscript{16}

\textsuperscript{14}The Appendix reports summary statistics from our sample. For all analysis in the paper, we apply the sample weights in each survey.

\textsuperscript{15}The 67 occupations are based on the occupational classifications in the 1990 Census. See \url{http://www.bls.gov/nls/quex/r1/y97r1cbka1.pdf}. We chose the 1990 occupation codes because they are available in all Census and ACS years since 1960. Appendix Table C2 reports the 67 occupations we analyze. Some examples of the occupational categories are “Executives, Administrators, and Managers”, “Engineers”, “Natural Scientists”, “Health Diagnostics”, “Health Assessment”, and “Lawyers and Judges.” We have also experimented with a more detailed classification of occupations by using 340 three digit occupation groupings that were defined consistently since 1980, as well as aggregating occupations into 20 broad occupational groups defined consistently since 1960. Our results were broadly similar at these different levels of occupation aggregation.

\textsuperscript{16}Our results were not altered when adjusting for hours worked across groups. This is not surprising given that we already condition on full time work status. When computing average earnings by occupation, we include both top-coded and imputed data. We experimented with excluding top-coded and imputed data and it had no effect on our estimated $\tau$’s.
Figure 1: Standard Deviation of Relative Occupational Shares

Note: Figure shows the standard deviation of $\ln\left(\frac{p_{wm}}{p_{wmen}}\right)$ across occupations for each group weighting each occupation by the share of earnings in that occupation. Specifically, we show the data for young white women (middle line), young black men (bottom line), and young black women (top line) relative to young white men.

3.2 Composite Frictions vs. Occupational Preferences

Equation 4 says that differences in occupational choice between women and blacks relative to white men are driven by differences in the ratio of occupational preferences to occupational frictions $\tilde{z}/\tau$. Figure 1 plots the standard deviation of the occupational shares of women and blacks relative to white men across market occupations for the young cohort in each decade. As can be seen, occupational sorting of women and blacks have converged to that of white men over time. Viewed through the lens of equation (4), this fact indicates that either $\tau$ and/or $\tilde{z}$ of women and blacks must have converged to that of men. This is one key fact behind our finding that the allocation of talent has improved over the last five decades.

Equations (7) and (8) show how we determine how much of the convergence in the occupational distribution is driven by $\tau$ vs. $\tilde{z}$. Equation (8) says that wage gaps across occupations for the young are proportional to gaps in $\tilde{z}^{-\frac{1}{1-\nu}}$. For example, if white women are poorly compensated (relative to white men) as lawyers compared
to secretaries, it must be the case that women receive higher utility from working as secretaries compared to lawyers. When $\delta = 0$, conditional on an estimate of $\eta$, we can exactly infer relative $\tilde{z}$ across groups by fitting the occupational wage gaps across groups and occupations for the young.

Equation (7) then says that, conditional on having an estimate of the parameters $\theta$ and $\eta$, the composite friction $\tau$ can be recovered from data on relative occupational shares after controlling for the average wage gap. Intuitively, the wage gap controls for the effect of preferences on occupational choice. The “residual” occupational choice is therefore only driven by the effect of $\tau$. Our base results normalize $\tilde{h}_{ig}/\tilde{h}_{i,wm} = 1$ and assume the occupational choice of white men is undistorted (i.e., $\tau_{i,wm} = 1$).\textsuperscript{17} So when the share of some group in an occupation is low relative to white men after we control for the wage gap, we infer that the group faces a high $\tau^w$ or a high $\tau^h$ in the occupation.

We need estimates of $\theta$ and $\eta$ to infer $\tau$’s and $\tilde{z}$’s from the data. To estimate $\theta$, we use the fact that distributional assumptions imply that wages within an occupation for a given group follow a Fréchet distribution with the shape parameter $\theta(1 - \eta)$. This reflects both comparative advantage (governed by $1/\theta$) and amplification from endogenous human capital accumulation (governed by $1/(1 - \eta)$). Using micro data from the U.S. Population Census/ACS, we estimate $\theta(1 - \eta)$ to fit the distribution of the residuals from a cross-sectional regression of log hourly wages on 66x4x3 occupation-group-age dummies in each year.\textsuperscript{18} The resulting estimates for $\theta(1 - \eta)$ range from a low of 1.24 in 1980 to a high of 1.42 in 2000, and average 1.36 across years.\textsuperscript{19}

The parameter $\eta$ denotes the elasticity of human capital with respect to education spending and is equal to the fraction of output spent on human capital accumulation. Spending on education (public plus private) as a share of GDP in the U.S. averaged 6.6 percent over the years 1995, 2000, 2005, and 2010.\textsuperscript{20} Since the labor share in the U.S. in the same four years was 0.641, this implies an $\eta$ of 0.103.\textsuperscript{21} With our base estimate of $\theta(1 - \eta) = 1.36$, $\eta = 0.103$ gives us $\theta = 1.52$.

\textsuperscript{17}We will later show robustness to these two normalizations.
\textsuperscript{18}We use MLE, with the likelihood function taking into account the number of observations which are top-coded in each year.
\textsuperscript{19}Sampling error is minimal because there are 300-400k observations per year for 1960 and 1970 and 2-3 million observations per year from 1980 onward. We did not use 2010 data because top-coded wage thresholds differed by state in that year.
\textsuperscript{21}Labor share data are from https://research.stlouisfed.org/fred2/series/LABSHPUSA156NRUG. The young’s share of earnings is from the U.S. Population Census/ACS.
Alternatively, we can estimate $\theta$ from the elasticity of labor supply. In our model, the extensive margin elasticity of labor supply with respect to a wage change is $\theta (1 - LFP_g)$. The meta analysis in Chetty et al. (2012) suggests an extensive margin labor supply elasticity of about 0.26 for men. The underlying data in their meta analysis come from the 1970-2007 period. In 1990, roughly in the middle of their analysis, 89.9% of men aged 25–34 were in the labor force. To match a labor supply elasticity of 0.26, our model implies that $\theta$ would equal 2.57. This is higher than the estimate of $\theta$ we get from wage dispersion. As a compromise between our two estimates, we will use $\theta = 2$ as our base case, but will also provide results with $\theta$ of 1.5 and 4.

With these values for $\theta$ and $\eta$, we can now infer $\tau$ and $\tilde{z}$ from data on occupational propensities and wage gaps. Figure 2 shows the mean of $\tau$ of each group across the 67 occupations. For white women, the mean of $\tau$ fell from about 7 in 1960 to around 3 in 2010, with most of the decline occurring prior to 1990. Average $\tau$ facing black women declined from around 8 to about 3 from 1960 through 2010. Black men experienced a decline in mean $\tau$ from around 3 to 1.5 during the five decades. For both black women and black men, most of the decline occurred between 1960 and 1980.

Figure 3 shows the dispersion of $\ln \tau$ (left panel) and $\ln \tilde{z}$ (right panel) across all 67 occupations. For all three groups, the variance of $\ln \tau$ fell by about 0.4 log points be-
Figure 3: Variance of Composite Frictions and Occupational Preferences

Note: Figure shows the earnings-weighted variance of ln $\tau$ (left panel) and ln $\tilde{z}$ (right panel).

tween 1960 and 2010. The right panel shows that the decline in the dispersion of ln $\tilde{z}$ is much smaller than the decline in the dispersion of $\tau$. For black men and white women, there is essentially no change in the dispersion of occupational preferences relative to white men so almost all of the occupational convergence is due to $\tau$. So for black men and white women, almost all of the convergence in occupational propensities is due to the convergence in $\tau$. For black women, the variance of relative ln $\tilde{z}$ fell, but the magnitude of the decline is only 17% of the decline in the dispersion of ln $\tau$. So even for black women, most of the occupational convergence is due to $\tau$ convergence.

Figure 4 displays $\tau$ for white women for a subset of occupations. The composite friction was high for women in 1960 working in construction, as lawyers, and as doctors relative to working as teachers and secretaries. For white women lawyers and doctors, $\tau$ in 1960 was around 10. If $\tau$ reflected labor market discrimination only, the implication would be that women lawyers in 1960 were paid only one-tenth of their marginal product relative to their male counterparts. The model infers large $\tau$’s for white women in these occupations in 1960 because there were few white women doctors and lawyers in 1960, even after controlling for the gap in wages. Conversely, a white woman in 1960 was 24 times more likely to work as a secretary than was a white man. The model explains this huge gap by assigning a $\tau$ below 1 for white women secretaries.

Over time, $\tau$ of white women in the lawyer and doctor professions fell. By 2010,
white women faced composite frictions below 2 in the lawyer, doctor, and teacher occupations. The barrier facing white women in the construction sector remained large. This fact could be the result of women having a comparative disadvantage (relative to men) as construction workers, a possibility we consider later in our robustness checks.

### 3.3 Labor Market vs. Human Capital Discrimination

The occupational frictions shown in Figures 3 and 4 are a composite of labor market discrimination ($\tau^w$) and human capital barriers ($\tau^h$). We now show how we distinguish between these two forces by exploiting life-cycle variation. The key assumption is that individuals make an active choice to obtain human capital prior to entering the labor market. This assumption implies that human capital discrimination is akin to a cohort effect, whereas labor market discrimination affects all cohorts in the labor market at the same point in time and thus is like a time effect.

The wage gap of cohort $c$ and group $g$ (relative to white men) in occupation $i$ at time $t$ relative to the wage gap at time $c$ (when cohort $c$ was young) is

$$\frac{\text{gap}_{ig}(c, t)}{\text{gap}_{ig}(c, c)} \propto \frac{1 - \tau^w_{ig}(t)}{1 - \tau^w_{ig}(c)}$$

(14)
The change in the wage gap over the lifecycle depends on the change in $\tau^w$ over time. If labor market discrimination diminishes over time, this raises the average wage (relative to white men) in occupations where the group previously faced discrimination. We therefore use the change in the wage gap over a cohort-group’s lifecycle to infer the change in $\tau^w$ over time. We then use $\tau_{tg} \equiv (1 + \tau^h_{tg})\eta/(1 - \tau^w_{tg})$ to infer the change in $\tau^h$ from the change in $\tau$ after controlling for the change in $\tau^w$. Intuitively, the change in $\tau^h$ is calculated as the difference in the wage gap of the young between successive cohorts after controlling for the slope of the life-cycle wage gap for a given cohort.

Figure 5 shows the wage gap of white women (left panel) and black men (right panel) vis a vis white men data for different cohorts over their life cycle. A decline in $\tau^w$ in a given year steepens a given life cycle profile. As $\tau^w$ falls, the wage of a given group relative to white men converges during an individual’s life cycle. On the other hand, a decline in $\tau^w$ and in $\tau^h$ across cohorts will shift up the intercept of the life cycle wage gap profiles. Figure 5 shows clearly the large increases in the intercept of the wage gap of the young white women across successive cohorts. However, there are only small changes in the slopes of the cohort profiles over time, which suggests that most of the shift of the intercept is due to a decline in human capital barriers. For black men, however, we see both shifts in the intercepts and steeping slopes particularly during the 1960s to 1980s suggesting a role for both declining human capital and labor market frictions.\footnote{We weight equally wage growth from young to middle age and from middle age to old to infer the change in $\tau^w$. We also need to normalize the initial split of the composite $\tau$ between $\tau^w$ and $\tau^h$. For our baseline, we assume a split of 50/50 in 1960. In subsequent years, we let the data speak to the importance of $\tau^h$ relative to $\tau^w$. Finally, we place an additional constraint on the $\tau$ breakdown to keep aggregate “revenue” from changing by more than 10 percent of GDP over our sample period. This requires $\tau^h$ to be no lower than 0.8, to keep subsidies for women secretaries from getting too large.}

### 3.4 Home Sector, Technology, and Return to Skill Parameters

We now show how we pin down the parameters that determine the labor force participation rate. Remember that the home sector is simply another sector so the labor force participation rate is simply determined by the returns in the market sector relative to the returns in the home sector. We assume the home sector is undistorted in that $\tau^h$ and $\tau^w$ in the home sector are zero for all groups. This implies that distortions in the market sector lowers the labor force participation rate. We also normalize the common home sector preference term ($\tilde{z}_{home}$) to 1 for all groups. This implies that the
Figure 5: Wage Gaps Relative to White Men by Time and Cohort

White Women

Black Men

Note: Log wage gaps are shown for the life cycle of each cohort by connected line segments for young, middle-aged, and old periods.

Figure 6: Mean of Occupational Preferences (Relative to White Men)

Note: Figure shows earnings-weighted mean of $\tilde{z}$ for each group relative to white men.

$\tilde{z}_i$'s we estimate for the market occupations are relative to the home sector. So the labor force participation rate is decreasing in $\tau^w$, $\tau^h$, $\tilde{z}$ and in the ratio of $w_i$ and $\phi_i$ in the home sector relative to the market sectors. And the gap in labor force participation rate relative to white men is decreasing in $\tau$ and $\tilde{z}$ ($w_i$ and $\phi_i$ have the same effect on labor force participation for all groups).
In Figure 3, we showed the dispersion of $\tilde{z}$ imputed from the dispersion of the wage gap across occupations, but not the level of $\tilde{z}$. We now use equation (5) to infer the level of $\tilde{z}$ (relative to white men) from the gap in the labor force participation rate after conditioning on the wage gap. Intuitively, the wage gap captures the effect of $\tau$ on labor force participation, so the residual has to be driven by preferences for market work relative to the home sector. Figure 6 shows that mean $\tilde{z}$ for white women was about 0.5 in 1960. Intuitively, the gap in labor force participation rates between white women and men in 1960 was larger than can be explained by the wage gap, so we infer that white women did not like working in the market sector relative to the home sector in 1960. Over time, to match the fact that female labor force participation rates rose relative to that of white men, the model infers that white women's preference for working in the market sector relative to the home sector must have increased. For black men in 1960, the gap in labor force participation rates relative to white can be entirely explained by the wage gap, so average $\tilde{z}$ is about 1. Over time, the labor force participation rate of black men fell relative to white men. The model “explains” this fact as a result of a decline in the mean $\tilde{z}$ of black men in the market sectors relative to white men.

The last thing are the parameters that determine the level of the labor force participation rates of white men, which are $w_i$ and $\phi_i$ in all sectors (including the home sector) and $\tilde{z}$ in the market sectors for white men. We pick $\phi_i$ in each year to match data on schooling differences for young white men across occupations in each year.\(^{23}\) Then, conditional on estimates of $\phi_i$, we pick $\tilde{z}_i$ in each occupation to fit the average wage by occupation. Then, given $\tilde{z}_i$ and $\phi_i$, we pick $w_i$ to exactly fit occupational shares for young white men. Occupations with a large share of young white men in a given year are ones where the price of skills $w_i$ is high. With estimates of $w_i$, we then back out the technology parameter $A_i$.\(^{24}\)

\(^{23}\)The first order conditions for schooling (equation 3) says that $s_i$ is a function of $\phi_i$ and the parameters $\eta$ and $\beta$. We assume the pre-market period is 25 years long so that $s_i = \frac{\text{Years of Education}}{25}$. We already have an estimate of $\eta$ so all we need is $\beta$. The average wage of group $g$ in occupation $i$ is proportional to $e^{\beta \psi}$ so the Mincerian return $\psi$ at 1 year around mean schooling $\bar{s}$ satisfies $e^{\beta \psi} = \left(\frac{1 - \bar{s} + 0.04}{1 - \bar{s} - 0.04}\right)^{\frac{1}{\beta}}$ so $\beta = \frac{\ln \left(\frac{1 - \bar{s} + 0.04}{1 - \bar{s} - 0.04}\right)}{(6 \psi)}$. Since the average Mincerian return from a cross-sectional regression of log income on years of schooling (with group dummies) averages 12.7% in our data, this gives us $\beta = 0.231$.

\(^{24}\)We need the elasticity of substitution among occupations (in aggregating to final output) $\sigma$ to infer $A_i$ from $w_i$. We choose $\sigma = 3$ as our baseline value, but we have no information on this parameter. Given this, we explore the robustness of our results to alternate values of $\sigma$. 
Table 1: Identifying Assumptions and Normalizations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Determination</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{i,wm}$</td>
<td>Human capital barriers (white men)</td>
<td>Assumption</td>
<td>0</td>
</tr>
<tr>
<td>$\tau_{w}$</td>
<td>Labor market barriers (white men)</td>
<td>Assumption</td>
<td>0</td>
</tr>
<tr>
<td>$\bar{h}_{i,g}$</td>
<td>Talent in each occupation (all groups)</td>
<td>Assumption</td>
<td>1</td>
</tr>
<tr>
<td>$\tau_{h,home,g}$</td>
<td>Home human capital barriers (all groups)</td>
<td>Assumption</td>
<td>0</td>
</tr>
<tr>
<td>$\tau_{w,home,g}$</td>
<td>Home labor market barriers (all groups)</td>
<td>Assumption</td>
<td>0</td>
</tr>
<tr>
<td>$\tilde{z}_{home,g}$</td>
<td>Home occupational preference (all groups)</td>
<td>Normalization</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2: Baseline Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Determination</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>Fréchet shape</td>
<td>Wage dispersion, Frisch elasticity</td>
<td>2</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Goods elasticity of human capital</td>
<td>Education spending</td>
<td>0.103</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>EoS across occupations</td>
<td>Arbitrary</td>
<td>3</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Consumption weight in utility</td>
<td>Mincerian return to education</td>
<td>0.231</td>
</tr>
</tbody>
</table>

3.5 Recap and Model Fit

Table 1 summarizes the identifying assumptions and normalizations for our base parameterization of the model assuming all individuals only draw occupational talent ($\delta = 0$). To reiterate our normalizations, we assume $\tau_w$ and $\tau_h$ is zero for white men in all occupations. We can not identify the level of $\tau_w$ and $\tau_h$, only their levels relative to a given group. We also assume that relative innate talent levels are the same across all groups and are normalized to 1 ($\bar{h}_{i,g} = 1$). We also assume $\tau_w$ and $\tau_h$ in the home sector is zero for all groups. We normalize preferences in the home sector to be 1 for all groups. Again, we can not identify the level of preferences, but only their level relative to the home sector.

Table 2 summarizes the key parameters and Table 3 the endogenous variables and the target data for their indirect inference. Some forcing variables depend on cohorts.
Table 3: Forcing Variables and Empirical Targets when \( \delta = 0 \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Empirical Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_i(t) )</td>
<td>Technology by occupation</td>
<td>Occupations of young white men</td>
</tr>
<tr>
<td>( \phi_i(t) )</td>
<td>Time elasticity of human capital</td>
<td>Education by occupation, young white men</td>
</tr>
<tr>
<td>( \tau^h_{ig}(c) )</td>
<td>Human capital barriers</td>
<td>Occupations of the young, by group</td>
</tr>
<tr>
<td>( \tau^w_{ig}(t) )</td>
<td>Labor market barriers</td>
<td>Life-cycle wage growth, by group</td>
</tr>
<tr>
<td>( \tilde{z}_{ig}(c) )</td>
<td>Occupational preferences</td>
<td>Wages by occupation for the young</td>
</tr>
<tr>
<td>( \gamma(1), \gamma(2) )</td>
<td>Experience terms</td>
<td>Age earnings profile of white men</td>
</tr>
</tbody>
</table>

Note: The variable values are chosen jointly to match the empirical targets. \( \delta = 0 \) refers to the polar case where individuals draw idiosyncratic ability for each occupation, and there is no idiosyncratic heterogeneity in tastes across occupations.

and some on time, but never both. Variables changing by cohort include the human capital barriers \( (\tau^h) \), common group specific occupational preferences \( (z) \), and the elasticity of human capital with respect to time investment \( (\phi) \). Labor market barriers \( (\tau^w) \) and technology parameters \( (A) \) vary over time. Human capital barriers, labor market discrimination, and occupational preferences vary across occupation-groups.

Finally, Table 4 compares the data and the model’s predictions for aggregate earnings per worker and labor force participation by year. Remember that the model only targets the occupational shares and labor force participation rates of the young. Despite this, predicted per-capita earnings and labor force participation rates in the model are not very far from the data. For example, in 2010 predicted earnings in the model is within 2 percent of the actual earnings in the data. In the model, labor force participation rate increases by 15 percentage points between 1960 and 2010. The actual increase between 1960 and 2010 is 16 percentage points.

4 Results with Selection Only on Ability \( (\delta = 0) \)

Given the discussion of inference above, we can now answer the key question of the paper: how much of the overall growth from 1960 to 2010 can be explained by the changing labor market outcomes of blacks and women during this time period? In
Table 4: Model versus Data: Earnings and Labor Force Participation

<table>
<thead>
<tr>
<th>Year</th>
<th>Earnings Data</th>
<th>Earnings Model</th>
<th>LFP Data</th>
<th>LFP Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>18,383</td>
<td>18,615</td>
<td>0.599</td>
<td>0.599</td>
</tr>
<tr>
<td>1970</td>
<td>24,645</td>
<td>25,000</td>
<td>0.636</td>
<td>0.614</td>
</tr>
<tr>
<td>1980</td>
<td>27,088</td>
<td>27,900</td>
<td>0.702</td>
<td>0.653</td>
</tr>
<tr>
<td>1990</td>
<td>33,953</td>
<td>34,265</td>
<td>0.764</td>
<td>0.720</td>
</tr>
<tr>
<td>2000</td>
<td>39,419</td>
<td>41,134</td>
<td>0.747</td>
<td>0.743</td>
</tr>
<tr>
<td>2010</td>
<td>41,541</td>
<td>42,717</td>
<td>0.759</td>
<td>0.748</td>
</tr>
</tbody>
</table>

Note: This table shows average market earnings per worker in 2009 dollars and labor force participation in the Census/ACS data alongside the corresponding model values by year.

In this section, we explore this question assuming that all individuals only draw occupation specific talent. In the next section, we explore how the results would change if individuals also draw occupational preferences.

Real earnings per person in our census sample grew by 1.8 percent per year between 1960 and 2010. According to our model, this observed earnings growth can come from five sources. First, growth in per capita earnings comes from general occupational productivity growth (changing $A$’s). Second, earnings growth results from growth in the returns to schooling resulting in more human capital attainment (changing $\phi$’s). Third, changing in preferences for each occupation including the home sector can reallocate labor across occupation resulting in earnings growth (changing $\tilde{z}$’s). Fourth, growth in the relative share of each group in the working age population can also mechanically result in changing earnings per capita (changing $q$’s). Finally, as described in Section 2.9, changing gender and race specific barriers to occupational choice in both the labor market and human capital market can result in economic growth (changing $\tau$’s).

The goal of our model is to assess how much of economic growth can be attributed to the changing $\tau$’s. We answer this key economic question by holding $\tau$’s fixed while allowing the $A$’s, $\phi$’s, $\tilde{z}$’s and $q$’s to evolve. For each variable, we calculate the difference between the actual path in the data and the counterfactual “no change in $\tau$’s” path to gauge the effect of changing $\tau$’s.
Table 5: Share of Growth due to Changing Frictions (all ages)

<table>
<thead>
<tr>
<th>Share of growth accounted for by</th>
<th>$\tau_h$ and $\tau_w$</th>
<th>$\tau_h$, $\tau_w$, $\tilde{\tau}$</th>
<th>$\tau_h$ only</th>
<th>$\tau_w$ only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market GDP per person</td>
<td>41.5%</td>
<td>40.8%</td>
<td>36.0%</td>
<td>7.7%</td>
</tr>
<tr>
<td>Market earnings per person</td>
<td>38.4%</td>
<td>37.5%</td>
<td>18.9%</td>
<td>26.0%</td>
</tr>
<tr>
<td>Labor force participation</td>
<td>90.4%</td>
<td>112.7%</td>
<td>24.9%</td>
<td>56.2%</td>
</tr>
<tr>
<td>Market GDP per worker</td>
<td>24.0%</td>
<td>15.0%</td>
<td>40.0%</td>
<td>-9.8%</td>
</tr>
<tr>
<td>Home+market GDP per person</td>
<td>32.7%</td>
<td>32.1%</td>
<td>30.6%</td>
<td>4.4%</td>
</tr>
</tbody>
</table>

Note: Entries in the table show the share of growth in the model attributable to changing frictions under various assumptions. The variables are $\tau_h$ (human capital frictions), $\tau_w$ (labor market frictions), and $\tilde{\tau}$ (occupational preferences).

4.1 Income and Productivity Gains

The results of our counterfactuals are shown in the first column of Table 5. The changes in $\tau$’s account for 41.5% of growth from 1960 to 2010 in market GDP per person (row 1) and 38.4% in market earnings per person (row 2). Market earnings and market GDP differ due to changing “revenue” from labor market discrimination over time. Figure 7 shows how such revenue evolves from 1960 to 2010. The figure also displays how revenue from both $\tau_w$ and $\tau_h$ combined shrinks from around 4% of GDP in 1960 to -4% of GDP in 2010. This decline results in the market earnings growth of workers being slightly larger than market GDP growth over the sample.

A portion of the growth in both market GDP per person and market earnings per person reflects rising labor force participation of women in response to falling discrimination and barriers to human capital attainment. Aggregate labor force participation rates rose steadily in the data, from 60% in 1960 to 76% in 2010, primarily due to increased female labor supply. Changes in the $\tau$’s account for 90% of this increase, as seen in row three of Table 5. Declining $\tau$’s also contributed to the growth in market GDP per person by raising the average wages of those who work in the market. As seen in row 4 of the table, declining $\tau$’s account for 24% of the increase in market GDP per worker. Declining labor market frictions allowed women and blacks to better exploit their comparative advantage reducing misallocation in the economy. Given that
the occupations with the highest $\tau$'s in 1960 were more likely to be high skilled occupations, the declining $\tau$'s resulted in women and blacks accumulating more human capital which also contributed to aggregate growth in market output per worker.

The final row of column 1 of Table 5 shows that the declining $\tau$'s explain about one-third of the growth in total GDP (inclusive of home sector output) during the last fifty years. The reduction in labor market discrimination and barriers to human capital growth drew more women into the market sector which had a direct effect of raising market GDP per person and simultaneously lowering home sector GDP per person. On net, however, declining labor market frictions for women and blacks substantially increased the sum of market and home output per person.

Figure 8 shows the time series decomposition of growth in market GDP per person coming from the changing $\tau$'s. The top line in the figure shows growth in market GDP per person implied by the model. The bottom line is the counterfactual growth in market GDP if the $\tau$'s were held fixed. Not surprisingly, the productivity effect of the $\tau$'s have grown over time. Additionally, our results suggest that productivity growth
Figure 8: GDP per person, Data and Model Counterfactual

Note: The graph shows the cumulative growth in GDP per person (market), in the data (overall) and in the model with no changes in $\tau$’s as in Table 5.

would have been close to zero during the 1970s had it not been for the reduction in labor market barriers to blacks and women during that time period.

Column 2 of Table 5 assesses how much of growth can be explained by declining labor market and human capital frictions (the $\tau$’s) and changing common occupational preferences (the $\tilde{z}$’s). Comparing the first and second columns it can be seen that changing group specific occupational preferences has only a modest effect on productivity growth. If anything, changing occupational preferences actually had a slightly negative effect on market GDP per person. The combined effect of the $\tau$’s and $\tilde{z}$’s explained 40.5% of the growth in market GDP per person since 1960 whereas the $\tau$’s alone explained 41.5%. Changing preferences do explain a small amount of the change in aggregate labor force participation with most of the effect being driven by women. The results in column 2 also imply that the majority of the growth in market GDP per person over the last half century was due to changes in the $A_i$’s and the $\phi_i$’s. These forces are not group specific and explained 59% of the growth in market GDP per person between 1960 and 2010 ($1 - 0.408$).

Why can’t changing preferences for market work explain the growth in market GDP
per person? If women simply did not like some occupations in 1960, the model with only sorting on talent says they would have been paid more in occupations in which they were underrepresented. The data show no such patterns. The gender (wage) gap was no lower in skilled occupations, and it did not fall faster in skilled occupations as the share of women rose. So while preference changes did result in the reallocation of women and blacks across occupations and did explain some of the rise in their labor force participation, it did not generate substantial economic growth.

The last two columns of Table 5 report growth contributions from falling barriers to human capital accumulation ($\tau_h$ only) separately from falling labor market discrimination ($\tau_w$ only). Falling human capital barriers alone would have accounted for 36% of growth in market GDP per person, versus 8% from falling labor market discrimination. Falling labor market discrimination looms larger for growth in market earnings (26% of growth). The reason is that declining discrimination in the labor market contributes directly to earnings growth relative to output growth. When we look at growth in home plus market GDP, declining barriers to human capital are again more important (31% of growth) than diminishing labor market discrimination (4%). These results reinforce Figure 5, where most of the changes in the wage gap of white women relative to white show up more as a cohort effect rather than a time effect. Through the lens of our model, changes in the $\tau_h$'s show up only as a cohort effect while changes in the $\tau_w$'s show up as both a cohort and time effect.

Table 5 suggests falling labor market discrimination drove much (over 56%) of the rise in labor force participation. Falling barriers to human capital accumulation played a lesser role since human capital is also useful in the home sector, albeit less so than in some market occupations. The breakdown into contributions from human capital versus labor market barriers is also revealing for why the contribution to growth in market GDP per worker is smaller than the contribution to growth in market GDP per person (24% vs. 42%). Falling human capital barriers, on their own, would have explained 40% of the growth in market GDP per worker. But falling labor market discrimination actually lowered growth in market GDP per worker ($\tau_h$ only) by enticing workers with marginal talent to move out of the home sector and into market occupations.25

25In four of the five rows in Table 5, the combined effect of changing $\tau_h$ and $\tau_w$ is smaller than the sum of the effects from eliminating them individually. The explanation for this is that misallocation is convex in barriers. Reducing one of the barriers individually yields the largest gains to be had by moving highly misallocated workers to the right occupation.
Table 6 shows how the changing $\tau$’s (column 1) and combined $\tau$’s and $\tilde{z}$’s (column 2) affect earnings and wage gaps across groups. The third column shows the explanatory effect of our full model ($\tau$’s, $\tilde{z}$’s, $A$’s and $\phi$’s) on group specific earnings and wage gaps. A few things are of note from Table 6. First, our model collectively does very well in explaining the average earnings growth of all groups between 1960 and 2010. The model explains about 100% of earnings growth for black men and black women while only slightly overestimating earnings growth for white women and underestimating earnings growth for white men. Second, falling labor market frictions account for 77% of earnings growth for white women, 29% for black men, and 51% for black women. The declining $\tau$’s, particularly for women, were the primary source of earnings growth over the last half century. Third, Table 6 highlights that the changing $\tau$’s actually lowered wage growth of white men. This is because falling barriers to women and blacks in high skilled occupations caused white men to shift to lower wage occupations as these groups entered the occupations. For men (both black and white), wage growth was driven primarily by changes in technology and skill requirements ($A$’s and $\phi$’s). Finally, the changing $\tilde{z}$’s again only had only modest positive effects on the earnings growth of women and modest negative effects on the earnings growth on men. As women entered the labor force due to changing preferences this increased their market earnings and reduced the earnings of men.

Our model concludes that most of the change in wage gaps between groups and white men can be explained by falling $\tau$’s. Our model actually over-predicts the changing wage gaps for all groups. This is because the model slightly under-predicts the earnings growth of white men. With that in mind, declining $\tau$’s explain 148% of the declining wage gap of white women, while the model in total explains 177%. Other than this, our model does fairly well in predicting the changing wage gaps over time. Our model, collectively, over-predicts slightly the rising wages of women and blacks relative to white men during the 1960–2010 period. For women, the changing $\tau$’s more than explain the shrinking gender gap in wages observed in the data. Declining barriers to human capital attainment and declining labor market discrimination were primarily responsible for the declining gender and racial wage gaps during the last fifty years.

Table 7 breaks down the growth from changing $\tau$’s into contributions by each group. Changes in the $\tau$’s of white women were much more important than changes in the $\tau$’s
Table 6: Wage Gaps and Earnings by Group and Changing Frictions

<table>
<thead>
<tr>
<th></th>
<th>—— Share of growth accounted for by ——</th>
<th>Full Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau^h$ and $\tau^w$</td>
<td>$\tau^h$, $\tau^w$, $z$</td>
</tr>
<tr>
<td>Earnings, WM</td>
<td>-12.2%</td>
<td>-17.4%</td>
</tr>
<tr>
<td>Earnings, WW</td>
<td>76.9%</td>
<td>86.3%</td>
</tr>
<tr>
<td>Earnings, BM</td>
<td>28.7%</td>
<td>20.4%</td>
</tr>
<tr>
<td>Earnings, BW</td>
<td>50.6%</td>
<td>52.5%</td>
</tr>
<tr>
<td>Wage gap, WW</td>
<td>148.1%</td>
<td>98.3%</td>
</tr>
<tr>
<td>Wage gap, BM</td>
<td>98.0%</td>
<td>115.4%</td>
</tr>
<tr>
<td>Wage gap, BW</td>
<td>84.6%</td>
<td>71.4%</td>
</tr>
<tr>
<td>LF Participation</td>
<td>90.4%</td>
<td>112.7%</td>
</tr>
</tbody>
</table>

Note: Entries in the table show the share of growth in the model attributable to changing frictions and other variables. The frictions are $\tau^h$ (human capital) and $\tau^w$ (labor market), and $z$ are occupational preferences. The last column reports the share of observed growth explained by the full model solution, including the $A$ and $\phi$ variables.

of blacks in explaining growth in home+market output per person during the 1960-2010 period. This is primarily because white women are a much larger share of the population. Table 7 also shows that falling pre-labor market barriers to human capital accumulation contributed much more to growth did declining labor market barriers. However, as can be seen in Figure 5, declining human capital barriers and labor market discrimination were roughly equally as important for black men while for white women it was the decline in human capital barriers that was primarily important.

Finally, we can ask: how much additional growth could be achieved by reducing frictions ($\tau$’s) all the way to zero? If the remaining frictions in 2010 were removed entirely, we calculate that GDP today would be 9.9% higher. These remaining gains result from the fact that even in 2010 there are still some differences in occupational choice and average wages across groups. However, through the lens of our model, there are only modest potential gains in GDP from reducing the $\tau$’s for women and blacks fully to zero. Most of the large productivity gains from the occupational convergence across groups occurred between 1970 and 2000. This is one reason to be less optimistic
Table 7: Share of growth in Market GDP per person due to different groups

<table>
<thead>
<tr>
<th></th>
<th>1960–2010</th>
<th>$\tau^h$ and $\tau^w$</th>
<th>$\tau^h$ only</th>
<th>$\tau^w$ only</th>
</tr>
</thead>
<tbody>
<tr>
<td>All groups</td>
<td>41.5%</td>
<td>36.0%</td>
<td>7.7%</td>
<td></td>
</tr>
<tr>
<td>White women</td>
<td>33.8%</td>
<td>29.8%</td>
<td>6.1%</td>
<td></td>
</tr>
<tr>
<td>Black men</td>
<td>1.2%</td>
<td>0.7%</td>
<td>0.6%</td>
<td></td>
</tr>
<tr>
<td>Black women</td>
<td>3.7%</td>
<td>3.2%</td>
<td>0.9%</td>
<td></td>
</tr>
</tbody>
</table>

Note: Entries are the share of growth in GDP per person (home+market) from changing frictions for various groups over different time periods. The variables are $\tau^h$ (human capital barriers), and $\tau^w$ (labor market frictions).

about U.S. economic growth after 2010 compared to growth in the last half-century.

4.2 Model Gains vs. Back-of-the-Envelope Gains

Our baseline estimate in Table 5 suggests that $\tau^w$ and $\tau^h$ account for 42% of the gains in market GDP per person and 33% of the gains in total GDP per person. Is this number large or small relative to what one might have expected? We have two ways of thinking about this question. First, in the log-normal approximation to the model with only $\tau^w$ variation that we presented back in Section 2.9, the elasticity of GDP to $1 - \text{mean of } \tau^w$ is $q_w \cdot \eta (1 - \eta)$. If we assume that the share of women in the population $q_w$ is 1/2 and $\eta = 0.1$ then this elasticity is $\frac{1}{2} \cdot \frac{1}{3}$. Figure 2 showed that the mean of the composite $\tau$ of women fell from about 10 in 1960 to 3 in 2010. This decline in $\tau$ can thus account for a 7% increase in total GDP per person.$^{26}$ Figure 3 shows that $\text{Var } \ln \tau^w$ fell from about 0.9 to 0.6 from 1960 to 2010. In the log-normal approximation to the model, the semi-elasticity of GDP to $\text{Var } \ln \tau$ is $q_w \cdot \left( \frac{1}{2} \cdot \frac{\eta - 1}{1 - \eta} \right) \approx 0.3.$ A 0.3 decrease in the variance of $\ln \tau$ thus could explain an 8% increase in total GDP per person. Thus, according to this back-of-the-envelope calculation, changing $\tau$’s for women boosted GDP about 15%. A similar calculation for black men suggests that changing $\tau$ for black men boosted GDP by about 2%.$^{28}$ The overall increase of GDP per person in our setup

\[ \frac{1}{2} \cdot \ln(10/3) \approx 0.07. \]
\[ \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{2 - 1}{1 - 1/2} \approx 0.28. \]
\[ \frac{1}{2} \cdot \ln(10/3) \approx 0.07. \]
\[ \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{2 - 1}{1 - 1/2} \approx 0.28. \]
was about 138%, so the changing \( \tau \) explains \( 0.15/1.38 \approx 12.3\% \) of growth in total GDP. This is significantly lower than the 33% contribution we estimate without imposing any parametric assumption on the distribution of the barriers. Clearly, log-normal is not a reasonable assumption. Also, the back-of-the-envelope calculation assumes no general equilibrium effects on wages of white men from changes in the labor supply of women and blacks across occupations.

A second way to answer the question is to compare our model-based contribution in market earnings per person growth to what one would infer from the falling gaps in earnings per person for women and blacks relative to white men. The narrowing gaps in earnings per person — including both declining wage gaps and rising labor force participation — mechanically account for 37% of growth in market per person.\(^{29}\) Through the lens of the model, it is coincidental that the model based estimate of market earnings per person growth is roughly the same as the naive back-of-the-envelope estimate with both suggesting that roughly 40 percent of market growth comes from the better allocation of women and black men to occupations. The back-of-the-envelope calculation assumes no general equilibrium effect of falling frictions on the earnings growth of white men. Yet we reported above white men's wages fell 11% relative to what they would have done without the changing barriers facing women and blacks (see Table 6). Moreover, this back-of-the-envelope calculation assumes that earnings gaps would not have changed in the absence of falling frictions. That is, this calculation implicitly attributes the entire decline in earnings gaps to changing frictions.

As we also show in Table 6, other forces such as changes in occupational productivity and returns to schooling have also had an effect in explaining changing wage gaps between groups over time. Both of these effects go in opposite directions. It just so happens that these effects roughly cancel out such that our model predictions and the back-of-the-envelope predictions are roughly equal. However, this back of the envelope calculation also suggests that our model based estimates of the growth consequences from falling labor market barriers to women and black men are not implausibly large.

\(^{29}\)For this calculation, we held fixed earnings per person relative to white men at 1960 levels, and found only 63% as much growth in earnings per person as seen in the data.
4.3 Robustness to Alternate Values of $\theta$, $\eta$ and $\sigma$

Table 8 explores robustness of our productivity gains to different parameter values assuming all individuals only draw occupational talent ($\delta = 0$). For each set of parameter values considered, we recalculate the $\tau$, $\tilde{z}$, $A$, and $\phi$ values so that the model continues to fit the occupation shares, wage gaps, etc by group and year. The first row of Table 8 replicates the gains from changing the $\tau$’s in explaining the growth in market GDP per person under baseline parameter values for comparison. The next row considers a lower value for the Fréchet shape parameter $\theta$, which is inversely related to the dispersion of comparative advantage across occupations. With $\theta = 1.5$ rather than the baseline $\theta = 2$, changing barriers explain modestly more of growth in market GDP per person (45.3%) than in the baseline (41.5%).

Recall that our baseline $\theta$ was estimated from wage dispersion within occupation-groups. This might overstate the degree of comparative advantage because some of the wage variation is due to absolute advantage. We thus entertain a much higher value ($\theta = 4$) than in our baseline ($\theta = 2$). With this higher $\theta$, the share of growth from changing $\tau$’s falls to 32.2% (vs. 41.5% in the baseline). Less discrimination is needed to explain occupational choices when comparative advantage is weak. Even with this higher value of $\theta$, however, declining $\tau$’s explain about one-third of growth in market GDP per person over the last half-century.

Table 8 also varies $\eta$, the elasticity of human capital with respect to goods invested in human capital. Intuitively, the gains from falling human capital barriers are greater the higher is $\eta$: the gains rise slightly from 40% with $\eta = 0.05$ to 42% with our baseline $\eta = 0.103$ to 44% with $\eta = 0.20$. As seen from this variation, our results are quite robust to alternate values of $\eta$.

The last rows of Table 8 show the sensitivity of the results to the elasticity of substitution $\sigma$ between occupations in production. When the elasticity of substitution across occupations is higher, the declining $\tau$’s explain a higher portion of the growth in market GDP per person. The gains in market GDP per person to changing $\tau$’s when $\sigma = 1.05$ (close to Cobb-Douglas) is 22.2%. While it may appear our results are quite sensitive to changes in $\sigma$, it should be noted that changing $\sigma$ simply reallocates how much of the growth occurs in the market sector versus home sector. When $\sigma = 1.05$, the declining $\tau$’s explain 33.5% of the growth in total GDP per person (inclusive of the
Table 8: Robustness to Parameter Values

<table>
<thead>
<tr>
<th></th>
<th>Growth in GDP per person accounted for by</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau^h$ and $\tau^w$</td>
</tr>
<tr>
<td>Benchmark</td>
<td>41.5%</td>
</tr>
<tr>
<td>$\theta = 1.5$</td>
<td>45.3%</td>
</tr>
<tr>
<td>$\theta = 4$</td>
<td>32.2%</td>
</tr>
<tr>
<td>$\eta = 0.05$</td>
<td>39.8%</td>
</tr>
<tr>
<td>$\eta = 0.20$</td>
<td>44.4%</td>
</tr>
<tr>
<td>$\sigma = 1.05$</td>
<td>22.2%</td>
</tr>
<tr>
<td>$\sigma = 10$</td>
<td>42.9%</td>
</tr>
</tbody>
</table>

Note: Entries show the share of growth in the model attributable to changing frictions $\tau^h$ (human capital) and $\tau^w$ (labor market). The baseline parameter values are $\theta = 2$, $\eta = 0.103$, and $\sigma = 3$.

home sector). This is nearly identical to our base results in Table 5. However, when $\sigma = 1.05$, the declining $\tau$’s explain only 70% of the rise in labor force participation. With a lower elasticity of substitution, less workers migrate from the home sector to the market sector when labor market frictions fall.

The moderate sensitivity of our results to $\theta$, $\eta$ and $\sigma$ may seem puzzling. But remember that, as we entertain different parameter values, we simultaneously change the $A$’s and $\tau$’s to fit observed wages and employment shares of the young in each occupation and group in each year. However, our results are more sensitive if we vary the key parameters holding all other parameter values and forcing variables fixed (the $A$’s, $\phi$’s, $\tau$’s, etc.). That is, the sensitivity increases if we do not re-calibrate. Consistent with the intuition provided in Section 2.9, the gains from changing $\tau$’s rise dramatically as we raise $\theta$ holding the other forcing variables fixed. Specifically, when ability is less dispersed ($\theta$ is higher), comparative advantage is weaker and the allocation of talent is more sensitive to changing $\tau$’s. The higher is $\theta$, the more occupational decisions are distorted by given barriers, and hence the bigger the gains from removing them. For example, if we hold the $A$’s, $z$’s, $\tau$’s, and $\phi$’s fixed at baseline estimates, the declining $\tau$’s explain 177% of the growth in market GDP per person when $\theta = 3$. 
Table 9: Forcing Variables and Empirical Targets with $\delta = 1$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Empirical Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_i(t)$</td>
<td>Technology by occupation</td>
<td>Occupations of young white men</td>
</tr>
<tr>
<td>$\phi_i(t)$</td>
<td>Time elasticity of human capital</td>
<td>Education by occupation, young white men</td>
</tr>
<tr>
<td>$\tau_{ig}(c)$</td>
<td>Human capital barriers</td>
<td><strong>Wages by occupation for the young</strong></td>
</tr>
<tr>
<td>$\tau_{w}(t)$</td>
<td>Labor market barriers</td>
<td>Life-cycle wage growth, by group</td>
</tr>
<tr>
<td>$\tilde{z}_{ig}(c)$</td>
<td>Occupational preferences</td>
<td><strong>Occupations of the young, by group</strong></td>
</tr>
<tr>
<td>$\gamma(1), \gamma(2)$</td>
<td>Experience terms</td>
<td>Age earnings profile of white men</td>
</tr>
</tbody>
</table>

Note: The variable values are chosen jointly to match the empirical targets. $\delta = 1$ refers to the polar case where individuals draw idiosyncratic tastes for each occupation, and there is no idiosyncratic heterogeneity in ability across occupations.

5 Inference and Results with Idiosyncratic Preferences ($\delta > 0$)

5.1 Inference with $\delta = 1$

When selection is entirely on idiosyncratic preferences rather than idiosyncratic ability, the way we identify the $\tau$’s and $\tilde{z}$’s changes. Table 9 summarizes how, bolding the entries that switch in this polar case. With selection only on preferences, average preferences for an occupation do not affect average wages in that occupation. Using equation (7) and imposing $\delta = 1$, the average wage gap in an occupation between a group and white men can be expressed as:

$$\frac{\text{wage}_{ig}(c,c)}{\text{wage}_{i,wm}(c,c)} = \left[ \frac{\tilde{h}_{ig}/\tilde{h}_{i,wm}}{\tau_{ig}(c,c)/\tau_{i,wm}(c,c)} \right]^{1/(1-\eta)}$$

With $\delta = 1$ and $\tilde{h}_{ig}/\tilde{h}_{i,wm} = 1$, the wage gap between groups within an occupation is pinned down by the relative $\tau$’s between groups in that occupation. Conditional on estimating $\tau$’s from the wage gaps, the relative $\tilde{z}$’s can be inferred from the relative share of a group in that occupation. In the polar case ($\delta = 0$) considered earlier, the inference was precisely the opposite: the $\tilde{z}$’s were inferred from the occupational wage gaps and the $\tau$’s were inferred from the differences in occupational sorting (conditional on the $\tilde{z}$’s).
Figure 9: Wage Gaps vs Propensities across Occupations for White Women in 1980

Note: The figure shows the relationship between the (log) occupational earnings gap for white women compared to white men (both in the young cohort) and the relative propensity to work in the occupation for the two groups, $p_{i,ww}/p_{i,wm}$ in 1980. A simple regression line through the scatter plot in the top panel yields a slope coefficient of 0.01 with a standard error of 0.01.

5.2 Calibrating $\delta$

If selection is based at all on tastes for an occupation ($\delta > 0$) instead of only on ability ($\delta = 0$), then average wages in an occupation should be positively correlated with occupational shares — to the extent such shares are driven by technology variables $A$ and $\phi$ across occupations. Figure 9 shows there is no systematic relationship between the share of young white women relative to young white men in an occupation in 1980 and the corresponding occupational wage gaps between young women and men in the same year. For example, young white women were 64 times more likely to work as secretaries as were young white men in 1980 and were one-fourth as likely to work as a lawyer in 1980. Yet the wage gap between young white women and young white men among secretaries was nearly identical to the gender wage gap among lawyers. Figure 10 shows that there is similarly no systematic correlation between the change in relative gender occupational shares and the change in the gender occupational wage gap over that time, both from 1960 to 2010.

Of course, it is possible to reconcile any pattern of wage gaps — and how they

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30A weighted regression of the scatter plot yields a slope coefficient of 0.01 with a standard error of 0.01.
change over time — with just the right levels and changes in preferences ($\tilde{\tau}$’s) across occupation-groups and years. To offset selection on idiosyncratic preferences, common group specific occupational preferences (the $\tilde{\tau}$’s) would need to be negatively correlated with occupational barriers. Women would need to dislike working in occupations in which they are discriminated against. As occupational barriers fall, their preferences would need to move in favor of those occupations. In this sense, the $\tau$’s and $\tilde{\tau}$’s could both be barriers to occupational choice and human capital formation for women and blacks.

The gains from changing $\tau$’s and $\tilde{\tau}$’s, however, may depend on the value of $\delta$. We therefore wish to show how the growth contributions of changing $\tau$’s and $\tilde{\tau}$’s vary with $\delta$. Just as important, we would like to estimate a plausible value for $\delta$. Rearranging (6) for young white men in year $t$, we can express schooling-adjusted wages as:

$$\frac{\text{wage}}{[1-s_i(t)]^{-\frac{1}{2}}} \propto p_i,wm(t) \frac{\delta}{\theta(1-\eta)} \tilde{\tau} \frac{\theta}{1-\eta}.$$  

Letting $\Delta$ denote first differences, we obtain:

$$\frac{\Delta \text{wage}}{[1-s_i(t)]^{-\frac{1}{2}}} \propto p_i,wm(t) \frac{\delta}{\theta(1-\eta)} \Delta \tilde{\tau} \frac{\theta}{1-\eta}.$$  

Note: The figure shows the relationship between the change in (log) occupational earnings gaps for white women compared to white men (both in the young cohort) and the change in the relative propensity to work in the occupation for the two groups, $p_i,ww/p_i,wm$, between 1960 and 2010. A simple regression line through the scatter plot in the top panel yields a slope coefficient of 0.06 with a standard error of 0.05.
\[ \Delta \ln \left( \frac{\text{wage}_{i,wm}(t)}{1 - s_i(t)} \right) = \text{constant} + \frac{\delta}{\theta(1-\eta)} \Delta \ln p_{i,wm}(t) - \frac{1}{1-\eta} \Delta \ln \tilde{z}_{i,wm}(t). \]  

As (15) demonstrates, \( \delta > 0 \) implies that wages (conditional on years of schooling \( s_i \)) should be rising in occupations with rising propensities, controlling for preferences. There will be no such relationship between wage changes and propensity changes when \( \delta = 0 \), i.e., when there is selection only on ability. When there is selection on preferences, marginal workers with little preference for an occupation must be attracted by higher wages to enter growing occupations.

If we run OLS on (15) for young white men from 1960 to 2010, the implied \( \delta \) is 0.077 with a standard error of 0.055. Figure 11 plots the empirical counterpart of such an OLS regression. There is a simultaneity problem with running OLS, however, because of the preference shifters in the residual of equation (15). To the extent occupations are growing due to changing preferences, wage growth will be more muted or even below-average in rising occupations. We thus propose to instrument for changing \( p \)'s using the model-implied changes in occupation technology parameters (\( A \)'s). This is a valid instrument assuming the orthogonality condition:

\[ \Delta \ln A_{i,wm} \perp \Delta \ln \tilde{z}_{i,wm} \]

That is, we assume that changes in technology are uncorrelated with changes in preferences across occupations for young white men. We iterate on \( \delta \) until we obtain a value such that the implied growth rates of technology and preferences are uncorrelated with each other. This yields an estimate of \( \delta = 0.22 \). This is higher than the OLS estimate because it adjusts for the covariance between changing wages and changing preferences in (15). The fact that changes in occupational propensities and wages (adjusted for years of schooling) are little correlated for young white men suggests that individuals primarily sort to occupations based on their draws of occupational talent as opposed to their draws of occupational preferences.

\[ 31 \text{Data on occupational earnings and average schooling by occupation are from the Census. We convert years of schooling into } s \text{ by dividing by 25 years, our assumed pre-work time endowment. We use the benchmark values of } \theta = 2 \text{ and } \eta = 0.103 \text{ to calculate the implied } \delta \text{ from the OLS coefficient. If we run this OLS regression across occupations in a given year, the coefficient averages 0.123 across years, with standard errors of around 0.04.} \]
Figure 11: Changes in Wages vs. Propensities, Young White Men 1960-2010

Note: The figure shows the relationship between the change in (log) occupational earnings for young white men (adjusted for schooling) and the change in the log of their propensity to work in the occupation, $p_{i,wm}$, between 1960 and 2010.

5.3 Results with $\delta > 0$

Table 10 examines the contribution of falling barriers to growth when we entertain $\delta = 0.22$, and also $\delta = 1/2$ and $\delta = 1$. The first column repeats our results with $\delta = 0$ for comparison. With our estimated value of $\delta = 0.22$, the share of growth attributed to changing $\tau$'s falls modestly. The differences are even smaller if we consider the share of growth coming from changing $\tau$’s and $\tilde{z}$’s.

The growth contributions from changing $\tau$’s fall more markedly when we move to $\delta = 0.5$, but still remain economically important. For example, changing $\tau$’s account for 28% of growth in market GDP per person with $\delta = 0.5$ versus 42% when $\delta = 0$. When $\delta = 0.5$, the share of growth coming from the $\tau$’s and $\tilde{z}$’s combined falls less, from 41% to 34%. As $\delta$ increases, the growth effects attributed to changing $\tau$’s falls while the growth effect from changing $z$’s rise, leaving the total effect on growth from the changing $\tau$’s and $z$’s less affected. In terms of economic growth, it matters little whether women in 1960 faced labor market or human capital frictions in doctor and lawyer occupations or whether women in 1960 just did not like to be doctors or lawyers.

Only when we go all the way to pure selection on idiosyncratic preferences, $\delta = 1$,
Table 10: Allowing for Selection on Idiosyncratic Preferences

<table>
<thead>
<tr>
<th></th>
<th>Share of growth accounted for by $\tau^h$ and $\tau^w$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta = 0$</td>
</tr>
<tr>
<td>Market GDP per person</td>
<td>41.5%</td>
</tr>
<tr>
<td>Market earnings per person</td>
<td>38.4%</td>
</tr>
<tr>
<td>Labor force participation</td>
<td>90.4%</td>
</tr>
<tr>
<td>Market GDP per worker</td>
<td>24.0%</td>
</tr>
<tr>
<td>Home+market GDP per person</td>
<td>32.7%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Share of growth accounted for by $\tau^h$, $\tau^w$, and $\tilde{z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta = 0$</td>
</tr>
<tr>
<td>Market GDP per person</td>
<td>40.8%</td>
</tr>
<tr>
<td>Market earnings per person</td>
<td>37.5%</td>
</tr>
<tr>
<td>Labor force participation</td>
<td>112.7%</td>
</tr>
<tr>
<td>Market GDP per worker</td>
<td>15.0%</td>
</tr>
<tr>
<td>Home+market GDP per person</td>
<td>32.1%</td>
</tr>
</tbody>
</table>

Note: Entries in the table show the share of growth in the model attributable to changing frictions $\tau^h$ (human capital frictions), $\tau^w$ (labor market frictions), and $\tilde{z}$ (occupational preferences), for alternate values of $\delta$, the fraction of workers who draw occupation-specific preferences rather than ability. $\delta = 0$ is our baseline with selection based only on talent. $\delta = 1$ is the opposite pole, with selection based only on preferences.

Do growth contributions from changing $\tau$’s plummet. In this case, the contributions of changing $\tau$’s and $\tilde{z}$’s combined fall as well, but less dramatically. The robust conclusion from Table 10 is that changing barriers account for a significant component of overall growth if selection on ability is at least as important as selection on idiosyncratic preferences.

6 Robustness

In this section, we explore the robustness of our results to alternate identifying assumptions and data moments used to discipline the model.
Table 11: Robustness to Alternative Assumptions about Group Differences in Talent

<table>
<thead>
<tr>
<th></th>
<th>Market GDP per person growth accounted for by $\tau_h$ and $\tau_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>41.5%</td>
</tr>
<tr>
<td>No frictions in “brawny”</td>
<td>38.7%</td>
</tr>
<tr>
<td>occupations</td>
<td></td>
</tr>
<tr>
<td>No frictions in 2010</td>
<td>35.8%</td>
</tr>
</tbody>
</table>

Note: Entries are the share of market GDP per person growth in the model attributable to changing frictions $\tau_h$ (human capital) and $\tau_w$ (labor market). A key identifying assumption is that the any talent differences across groups, to the extent they exist, are constant over time. In our baseline specification, we assume that $\bar{h}_{ig} = 1$ for all occupations and all groups. In other words, that there are no innate talent differences between groups. The first row of the table recounts are baseline estimates. In the second row, we allow men and women to have different $\bar{h}$'s in “brawny” occupations. Specifically, we assume no gender specific $\tau$'s in these occupations. Instead, we allow the $\bar{h}$'s to evolve to exactly fit the quantity data for these occupations. “No frictions in 2010” (the third row) assumes that there are no frictions in 2010 for any group, so that differences in $\bar{h}_{ig}$ explain all group differences in that year; we then calculate $\tau$'s for earlier years assuming the mean value of the distribution of market skills in 2010 apply to earlier years. For the results in this table, we assume $\delta = 0$.

6.1 Alternative Identifying Assumptions

One key identifying assumption that underlies our estimation is that any innate talent differences between men and women are constant over time. Under this assumption, changes in occupational sorting and wage gaps between groups inform us about changes in the $\tau$'s and $\bar{z}$'s. In our base specification, we go even farther and assume there are no innate talent differences between group in any period ($\bar{h}_{ig} = 1$ for all $i$ and $g$ in all time periods). In this section, we explore alternative assumptions while still holding relative talent across groups fixed over time.

Table 11 shows how our results change with alternative assumptions about the evolution of $\bar{h}$ across groups within different occupations over time under our base scenario of $\delta = 0$. The first row of the table redisplays our baseline estimates for market GDP per person growth from Table 5. The second row relaxes the assumption that men and women draw from the same distribution of talent in all occupations. In particular, we consider the possibility that some occupations rely more on physical strength than

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32Given that the growth results from $\delta = 0$ are so similar to the growth results with our estimates of $\delta = 0.22$, we focus our robustness results on the $\delta = 0$ scenario.
others, and that this reliance might have changed over time because of technological progress. To see the potential importance of this story, we go to the extreme of assuming no frictions at all faced by women in any of the occupations where physical strength is arguably important (i.e., \( \tau_{ig}^h = \tau_{ig}^w = 0 \) for women in these occupations). These occupations include construction, firefighters, police officers, and most of manufacturing.\(^{33}\) We estimate differences in \( \bar{h}_{ig} \) for young women to fully explain their allocation to these occupations in 1960, 1970, \ldots, 2010. As shown in Table 11, the fraction of market GDP growth per person explained by changing frictions falls only slightly from 41.5% to 38.7% with this alternative identifying assumption. Our results are not sensitive to this alternative because most of the gains we attribute to changing \( \tau \)'s come from the rising propensity of women becoming lawyers, doctors, scientists, professors, and managers — occupations where physical strength is not important.

The last row in Table 11 makes a more extreme assumption. In this alternative, we allow all groups to have different levels of innate talent in all occupations. We assume, however, that these innate talent differences are constant over time. Specifically, we assume all group differences among the young in 2010 reflect talent rather than distortions. Specifically, we set the 2010 \( \tau \)'s to zero for all groups and all occupations and assume differences in \( \bar{h}_{ig} \) fully account for group differences in occupational choice among the young in 2010.\(^{34}\) We keep talent in prior years at the 2010 values for each group, but back out distortions in earlier years. In essence, this specification allows for arbitrary talent differences between men and women to fit the 2010 data. Under this more flexible alternative, eliminating the \( \tau \)'s in the earlier years still generates 36% of growth in market GDP per person. Thus our gains are not an artifact of assuming the allocation of talent was far from optimal in 2010.

These exercises highlight our key identifying assumption. What is important is not that different groups have the same level of innate talent in all occupations. Instead, what is important is that, whatever the talent differences are across groups, those talent differences remain constant over time. This assumption is particularly important for high skilled occupations like doctors and lawyers and less important for “brawny” occupations like construction workers.

\(^{33}\)Rendall (2010) classifies occupations based on the importance of physical strength, and we define brawny occupations for our analysis as those occupations in the top half of her brawny distribution.

\(^{34}\)We still normalize \( \bar{h}_{iwm} = 1 \) in 2010.
Another assumption that facilitates our identification is that white men face no labor market or human capital frictions. An alternative assumption might be that there was no discrimination in 1960 at all, but growing discrimination against men and in favor of women since then. If we assume women and men have the same mean talent, as we do in our baseline specification of $\delta = 0$, this would imply identical average wages and occupational distributions for women and men in 1960. This is something we do not observe in the data. Assuming relative talent stays constant over time, this alternative would also require women to earn increasingly more than men and be increasingly overrepresented in high skill occupations after 1960. All of these predictions are at odds with the patterns documented above. If men and women have the same level of innate talent, the data strongly reject the hypothesis that men have been increasingly discriminated against over time.

Another alternative would be to assume discrimination in favor of men and no discrimination against women in 1960, with the discrimination in favor of men abating over time. This would fit the facts on wages and sorting over time, and would imply falling misallocation. But it is not isomorphic to our baseline assumption. First, it would imply falling education spending by men over the decades. Second, it would entail huge subsidies for men that diminish over time. When we calibrate our baseline model with $\delta = 0$, we find that earnings of men must exceed their marginal product by orders of magnitude. The implied total subsidy to men would be multiples of 1960 GDP. Men must be paid massive subsidies in order to induce so many of them, relative to women, to choose high skilled occupations. Earnings would need to vastly exceed GDP in 1960, which of course we do not observe. Such an extreme outcome does not arise under our baseline assumption because no revenue is collected from qualified women who are driven out of occupations by discrimination.

Yet another alternative would be to assume — contrary to our presumption — that women are somehow innately less talented than men, supposedly explaining women’s lower wages and underrepresentation in skilled occupations in 1960. Rising discrimination in favor of women since 1960 might then account for the closing gaps between men and women. This hypothesis would entail rising misallocation and a drag on aggregate growth. Data on individual test scores suggests women are not less talented than men. The Armed Forces Qualifying Test (AFQT) was administered in both the
NLSY 1979 and the NLSY 1997. The NLSY tracks a sample of individuals who were 12-16 years old when the surveys started. The AFQT scores in the NLSY are very similar for men and women in both 1979 and 1997. According to these scores, women seem no less talented than men in their early teens. If we condition on working, women likewise have similar scores to men in both 1979 and 1997. If one believes the story of rising discrimination in favor of women, one would have expected the relative test scores of working women to fall along with their rising participation rates. AFQT scores do not support the hypothesis that women are innately less talented than men.

Collectively, these results suggest that alternate assumptions do not fit aspects of the data as well as our baseline. We therefore prefer our baseline assumption that women and blacks faced human capital and labor market frictions in 1960 relative to white men, and that these frictions fell over time.

### 6.2 Other Robustness

Table 12 explores an additional set of robustness exercises. For comparison, the first row repeats our benchmark results for share of market GDP per person growth explained by the changing $\tau$’s. The next two rows show that the productivity gains we estimate are not proportional to the gender and race wage gaps we fed into the model. We can halve the wage gaps in all years, or even eliminate them in all years, and the implied $\tau$’s still explain 37.5% or 33.5% of growth in market GDP per person, vs. 41.5% in the baseline. One reason is that misallocation of talent by race and gender can occur even if average wages across groups are similar. The misallocation of talent is tied to the dispersion in the $\tau$’s, whereas the wage gaps are related to both the mean and variance of the $\tau$’s. Another reason is that the wage gap for white women would have widened in the absence of the changing $\tau$’s due to changes in the $A$’s and $\phi$’s. A key take away from this exercise is that productivity gains from changing labor market discrimination and barriers to human capital accumulation cannot be gleaned from the changing wage gaps alone.

Another assumption we make in our base specification is that the returns to expe-

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35In 1979, the average normalized AFQT score was 54.3 for white men and 53.6 for white women. In 1997, the respective averages were 55.5 and 57.4.
36In 1979, the average normalized AFQT score for working white men was 51.8 and for working white men was 52.3. In 1997, the respective averages were 52.6 and 54.9.
Table 12: Additional Robustness

<table>
<thead>
<tr>
<th></th>
<th>Market GDP per person growth accounted for by</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau^h$ and $\tau^w$</td>
</tr>
<tr>
<td>Benchmark</td>
<td>41.5%</td>
</tr>
<tr>
<td>Wage gaps halved</td>
<td>37.5%</td>
</tr>
<tr>
<td>Zero wage gaps</td>
<td>33.5%</td>
</tr>
<tr>
<td>Half the return to experience</td>
<td>42.1%</td>
</tr>
<tr>
<td>2/3, 1/3 split of $\tau_{i,g}$ in 1960</td>
<td>39.2%</td>
</tr>
<tr>
<td>1/3, 2/3 split of $\tau_{i,g}$ in 1960</td>
<td>41.7%</td>
</tr>
<tr>
<td>No constraint on $\tau^h$</td>
<td>46.1%</td>
</tr>
</tbody>
</table>

Note: See notes to Table 5. The baseline splits $\tau$ in 1960 evenly into $\tau^h$ and $\tau^w$ in 1960, but not in future years. The baseline also constrains $\tau^h$ to be at most $-0.8$. The robustness of 2/3, 1/3 split of $\tau_{i,g}$ means that 2/3 of the initial $\tau_{i,g}$ is assigned to $\tau^w_{i,g}$ in 1960. Conversely, the robustness of 1/3, 2/3 split of $\tau_{i,g}$ means that 1/3 of the initial $\tau_{i,g}$ is assigned to $\tau^w_{i,g}$ in 1960.

Experience are constant across groups and occupations over time. We want to stress that allowing for general returns to experience is not adding much to our inference. The fourth row in Table 12 illustrates this point. Specifically, in this robustness exercise, we cut productivity growth over the life cycle (old/middle and middle/young) in half for each group. Such a change barely alters our baseline results.

The final three rows of Table 12 considers additional robustness checks. In our benchmark we split the composite $\tau_{i,g}$ in 1960 evenly into $\tau^w$ and $\tau^h$. Our procedure estimates changes in $\tau$‘s over time but we need to make an assumption on the initial split between $\tau^h$ and $\tau^w$ given that we have nothing to discipline this initial split in the data. If we put more weight (2/3) on $\tau^w$ we account for 39.2% of growth in market GDP per person, versus 41.5% in the baseline. If we put less weight (1/3) on $\tau^w$ we account for 41.7% of growth. Finally, our benchmark case constrains the values of $\tau^h$ to be no smaller than $-0.8$. If we put no constraint on how negative $\tau^h$ can get, we explain 46.1% of growth vs. 41.5% in the baseline. Absent this constraint, the estimation implies human capital subsidies that exceed GDP because of large subsidies for white women (e.g. as secretaries).
6.3 Further Model Implications

While our model is stylized in many respects, it is able to match at least two other important facts that were not targeted in the estimation: trends in female labor supply elasticities and cross-state variation in survey measures of racial discrimination. We briefly discuss how our model matches these facts. An extended discussion of these results can be found in the Online Appendix that accompanies the paper.

Using data from the Current Population Survey, Blau and Kahn (2007) estimate that there was a dramatic decline in female labor supply elasticity during the 1980–2000 period. Helpful for comparing with the predictions of our model, they report female labor supply elasticities specifically for 25-34 year olds. We compare the model’s implied labor supply elasticities — equal to $\theta(1 - LPF_y)$ — for young white women to the estimated labor supply elasticities reported in Blau and Kahn (2007). Using our baseline $\theta$, the model matches both the level and the trend in female labor supply elasticities well. Nothing in our model is calibrated to match either the level or the trend in labor supply elasticities for women. As discussed earlier, we estimated $\theta$ to match the labor supply elasticity of men in 1980. The fact that we can roughly match the level of the labor supply elasticity for young women in three different time periods suggests that our model is broadly consistent with empirical moments outside the ones we used to calibrate the model.

There are very few micro-based measures of discrimination to which we can compare our estimated $\tau$’s. One such exception is the recent work by Charles and Guryan (2008). Charles and Guryan (CG) use data from the General Social Survey (GSS) to construct a measure of the taste for discrimination against blacks by whites for every state. Pooling together survey questions from the mid 1970s through the early 1990s and focusing only a sample of white respondents, Charles and Guryan make indices of the extent of racial discrimination in each state. Using our baseline model, we compute an average measure of $\tau_{bm}$ relative to white men separately for each state combining our 1980 and 1990 data. There is a very strong cross-state relationship between our measures of $\tau_{bm}$ and the CG discrimination index. The adjusted R-squared of the simple scatter plot across states of our $\tau_{bm}$ and the CG discrimination index is 0.6. Places we identify as having a high $\tau_{bm}$ are the same places Charles and Guryan find as being highly discriminatory towards blacks based on survey data from the GSS. The findings
provide additional external validity that our procedure is measuring salient features of the U.S. economy over the last five decades.

7 Conclusion

How does discrimination in the labor market and barriers to the acquisition of human capital for white women, black men, and black women affect occupational choice? And what are the consequences of the altered allocation of talent for aggregate income and productivity? We develop a framework to tackle these questions empirically. This framework has three building blocks. First, we use a standard Roy model of occupational choice, augmented to allow for labor market discrimination, barriers to the acquisition of human capital and occupation-specific preferences. Second, we impose the assumption that an individual's talent or preferences in each occupation follows an extreme value distribution. Third, we embed the Roy model in general equilibrium to account for the effect of occupational choice on the price of skills in each occupation and to allow for the effect of technological change on occupational choice. We use synthetic cohort data measuring changes in relative occupational sorting and wage gaps across time to discipline our model. A key identifying assumption is that the distribution of innate talent across groups is constant over time.

We apply this framework to measure the changes in barriers to occupational choice facing women and blacks in the U.S. from 1960 to 2010. We find large reductions in these barriers, concentrated in high-skilled occupations. We then use our general equilibrium setup to isolate the aggregate effects of the reduction in occupational barriers facing these groups. Our baseline calculations suggest that falling barriers explain roughly 40% of aggregate growth in market GDP per person.

We also explore the robustness of our results to a variety of alternate specifications. In our baseline model, we assume that individuals are heterogeneous only in their draws of occupational talent. We show that our baseline results are rather robust to the assumption that instead individuals are only heterogeneous in their draws of occupational preferences. Even under this extreme assumption, we find that of one-fifth in U.S. market GDP growth can be explained by group specific labor market discrimination, group specific barriers to human capital attainment, or common group specific
occupational preferences. Much of the productivity gains come from drawing women and blacks into high skilled occupations. Whether women increased their propensity to become lawyers and doctors because of declining labor market frictions or because women started liking these occupations more have broadly similar growth implications. Now saying that, we estimate that occupational sorting based on talent draws is a much better fit of the data than occupational sorting based on preference draws.

It should be clear that this paper provides only a preliminary answer to these important questions. The general equilibrium Roy model we use is a useful place to start, but it is possible that a different framework can do a better job. We abstract from allowing for correlations between an individual's absolute advantage and their comparative advantage. Additionally, the ease with which our model can be matched to observable moments of the data is facilitated by our assumption that comparative advantage or preferences is distributed according to a Fréchet distribution. These assumptions have the benefits of tractability but may abstract from other important features of the data. Some structure is needed in order to assess how the substantive changes in occupational sorting across gender and race affected U.S. economic growth. We provided one such framework as a starting point. Our results suggest that the decline in occupational and human capital barriers to women and blacks was a very important source of growth to the U.S. economy and the leveling out of changes may be one reason why growth has slowed down. However, we realize that our model is only a launching off point to address these important questions and expect some of our assumptions used for tractability to be relaxed as the literature progresses.

References


A Derivations and Proofs

The propositions in the paper summarize the key results from the model. This appendix shows how to derive the results.

Proof of Proposition 1. Occupational Choice

The individual’s utility from choosing a particular occupation, \( U(\tau_{\text{ig}}, w_{i}, \epsilon_{i}, \mu_{i}) \), is proportional to \( \mu_{i}(\bar{\gamma}\tilde{w}_{\text{ig}}\epsilon_{i})^{\frac{1}{1-\eta}} \), where \( \tilde{w}_{\text{ig}} \equiv \frac{h_{\text{ig}}w_{i}e^{\alpha_{i}[1-(1-s)\tau_{\text{ig}}]}}{\tau_{\text{ig}}} \) and \( \bar{\gamma} \equiv 1+\gamma(2)+\gamma(3) \) is the sum of the experience terms. We first consider the occupation decision for individuals with ability heterogeneity (so no taste heterogeneity or \( \mu_{i} = 1 \)). For these people, the solution to the individual’s problem involves picking the occupation with the largest value of \( \tilde{w}_{\text{ig}}\epsilon_{i} \). To keep the notation simple, we will suppress the \( g \) subscript in what follows.

Without loss of generality, consider the probability that the individual chooses occupation 1, and denote this by \( p_{1} \). Then

\[
p_{1} = \Pr \left[ \tilde{w}_{1}\epsilon_{1} > \tilde{w}_{s}\epsilon_{s} \right] \quad \forall s \neq 1
= \Pr \left[ \epsilon_{s} < \tilde{w}_{1}/\tilde{w}_{s} \right] \quad \forall s \neq 1
= \int F_{1}(\epsilon, \alpha_{2}\epsilon, \ldots, \alpha_{M}\epsilon) d\epsilon, \tag{A1}
\]

where \( F_{1}(\cdot) \) is the derivative of the cdf with respect to its first argument and \( \alpha_{i} \equiv \tilde{w}_{i}/\tilde{w}_{1} \).

Recall that

\[
F(\epsilon_{1}, \ldots, \epsilon_{M}) = \exp \left[ \sum_{s=1}^{M} \epsilon_{s}^{-\theta} \right].
\]

Taking the derivative with respect to \( \epsilon_{1} \) and evaluating at the appropriate arguments
gives
\[ F_1(\epsilon, \alpha_2 \epsilon, \ldots, \alpha_M \epsilon) = \theta \epsilon^{-\theta - 1} \cdot \exp \left[ \dfrac{\bar{\alpha} \epsilon^{-\theta}}{\alpha} \right] \]  
(A2)
where \( \bar{\alpha} \equiv \sum_s \alpha_s^{-\theta} \).

Evaluating the integral in (A1) then gives
\[
p_1 = \int F_1(\epsilon, \alpha_2 \epsilon, \ldots, \alpha_M \epsilon) d\epsilon
= \frac{1}{\bar{\alpha}} \int \bar{\alpha} \theta \epsilon^{-\theta - 1} \cdot \exp \left[ \dfrac{\bar{\alpha} \epsilon^{-\theta}}{\alpha} \right] d\epsilon
= \frac{1}{\bar{\alpha}} \cdot \int dF(\epsilon)
= \frac{1}{\bar{\alpha}}
= \frac{1}{\sum_s \alpha_s^{-\theta}}
= \frac{1}{\sum_s \bar{\omega}_s^\theta}
\]
A similar expression applies for any occupation \( i \), so we have
\[
p_i = \frac{\bar{\omega}_i^\theta}{\sum_s \bar{\omega}_s^\theta} \tag{A3}
\]

We now consider individuals with taste heterogeneity (so no ability heterogeneity or \( \epsilon_i = 1 \)). These individuals pick the occupation with the largest value of \( \mu_i(\bar{\omega}_g)^{\frac{3\beta}{1-\eta}} \).
The probability the individual picks occupation 1 is now given by
\[
p_1 = \Pr \left[ \bar{\omega}_1^{\frac{3\beta}{1-\eta}} \mu_1 > \bar{\omega}_s^{\frac{3\beta}{1-\eta}} \mu_s \right] \forall s \neq 1
\]
The probability of picking occupation 1 is then given by
\[
p_1 = \frac{\bar{\omega}_1^{\frac{3\beta}{1-\eta} \omega}}{\sum_s \bar{\omega}_s^{\frac{3\beta}{1-\eta} \omega}}
\]
where \( \omega \) is the shape parameter of the Fréchet distribution for tastes. Using the assumption that \( \omega = \frac{\theta (1-\eta)}{3\beta} \), the probability of picking occupation \( i \) is still given by equation (A3).

**Proof of Proposition 2.** Geometric Average of Worker Quality
Efficiency units of labor of an individual of cohort $c$ in occupation $i$ at time $t$ is given by $h_i(c, t) = \bar{h}_i s(c) \phi_i(c) e_i(c).$ Using the results from the individual’s optimization problem, it is straightforward to show that

$$h_i(c, t) e_i = s_i(c) \phi_i(c) \gamma(t - c) \left( \frac{\eta s_i(c) \phi_i(c) w_i(c) (1 - \tau_i w(c)) \bar{h}_i \gamma}{1 + \tau_i h(c)} \right)^{\frac{\eta}{1 - \eta}} e_i^{\frac{1}{1 - \eta}}.$$

For individuals that sort on ability, the geometric average of efficiency units of labor in an occupation is given by

$$e \log[h_i(c, t)e_i | \text{choose } i] = s_i(c) \phi_i(c) \gamma(t - c) \left( \frac{\eta s_i(c) \phi_i(c) w_i(c) (1 - \tau_i w(c)) \bar{h}_i \gamma}{1 + \tau_i h(c)} \right)^{\frac{\eta}{1 - \eta}} e \log \left[ e_i^{\frac{1}{1 - \eta}} | \text{choose } i \right]. \tag{A4}$$

We need to compute $e \log \left[ e_i^{\frac{1}{1 - \eta}} | \text{choose } i \right]$. Let $e^*$ denote ability in the chosen occupation. We need to know the distribution of $e^*$ raised to some power. Let $y_i \equiv \bar{w}_i e_i$ denote the key occupational choice term. Then

$$y^* \equiv \max_i \{y_i\} = \max_i \{\bar{w}_i e_i\} = \bar{w}^* e^*.$$

Since $y_i$ is the thing we are maximizing, it inherits the extreme value distribution:

$$\Pr[y^* < z] = \Pr[y_i < z] \forall i$$

$$= \Pr[\epsilon_i < z/\bar{w}_i] \forall i$$

$$= F \left( \frac{z}{\bar{w}_1}, \ldots, \frac{z}{\bar{w}_M} \right)$$

$$= \exp \left[ -\sum_s \bar{w}_s^\theta z^{-\theta} \right]$$

$$= \exp \left\{ -m z^{-\theta} \right\}.$$

That is, the extreme value also has a Fréchet distribution, where $m \equiv \sum_s \bar{w}_s^\theta$.

Straightforward algebra then reveals that the distribution of $e^*$, the ability of people in their chosen occupation, is also Fréchet:

$$G(x) \equiv \Pr[e^* < x] \equiv \exp \left[ -m x^{-\theta} \right]$$
where $m^* \equiv \sum_{s=1}^{M} (\tilde{w}_s / \tilde{w}^*)^\theta = 1 / p^*$. 

Next, we need an expression for the expected value of the chosen occupation’s ability raised to some power. Let $\lambda$ be some positive exponent. Then,

$$
\mathbb{E}[\epsilon^\lambda] = \int_0^\infty \epsilon^\lambda dG(\epsilon^*)
$$

Recall that the “Gamma function” is $\Gamma(\alpha) \equiv \int_0^\infty x^{\alpha-1}e^{-x}dx$. Using the change-of-variable $x = \frac{1}{p^*}(\epsilon^* - \theta)$, one can show that

$$
\mathbb{E}[\epsilon^\lambda] = \left( \frac{1}{p^*} \right)^{\lambda/\theta} \int_0^\infty x^{-\frac{\lambda}{\theta}} e^{-x}dx
$$

Applying this result to our model, we have

$$
\mathbb{E}[\epsilon^{1-\eta} | \text{choose } i] = \left( \frac{1}{p_{ig}} \right)^{\frac{1}{\theta} \frac{1}{1-\eta}} \Gamma \left( 1 - \frac{\lambda}{\theta} \cdot \frac{1}{1-\eta} \right).
$$

Finally note that if $x \sim \text{Frechet}(\theta)$, then $\log x \sim \text{Gumbel}(1/\theta)$, and $\mathbb{E}[\log x] = \gamma_{\text{em}}$, where $\gamma_{\text{em}} \approx 0.5772$ is the Euler-Mascheroni constant. Applying this to the expression for $\mathbb{E}\log \left[ \epsilon_i^{1-\eta} | \text{choose } i \right]$ above, we have

$$
\mathbb{E}\log \left[ \epsilon_i^{1-\eta} | \text{choose } i \right] = \left( \frac{1}{p_{ig}} \right)^{\frac{1}{\theta} \frac{1}{1-\eta}} \overline{\Gamma}
$$

where $\overline{\Gamma} \equiv e^{\gamma_{\text{em}}(1-\theta)}$. Substituting this expression into equation (A4) yields the geometric mean of ability for individuals who sort on ability:

$$
e^{\mathbb{E}\log[h_i(c,t)\epsilon_i | \text{choose } i]} = \overline{\Gamma} s_i(c) \phi_i(t) \gamma(t-c) \left[ \left( \frac{\eta s_i(c) \phi_i(c) w_i(c) (1 - \tau_{w}^i(c)) \tilde{h}_i \gamma}{1 + \tau_{w}^i(c)} \right)^\eta \left( \frac{1}{p_{ig}} \right)^\frac{1}{\theta} \right]^{1-\eta}.
$$

The last thing we need is efficiency units of workers that sort on preferences. Remember $\epsilon_i = \Gamma^{1-\eta}$ where $\Gamma \equiv \Gamma \left( 1 - \frac{1}{\theta(1-\eta)} \right)$. Therefore quality is the same for all
individuals and given by:

\[ h_i(c, t) \epsilon_i \text{ choose } i = \Gamma s_i(c) \phi_i(t) \gamma(t - c) \left( \frac{\eta s_i(c) \phi_i(c) w_i(c)(1 - \tau_i^m(c)) \bar{h} \tilde{\gamma}}{1 + \tau_i^h(c)} \right) \frac{r}{\eta} \]  

(A6)

Finally, the geometric average of the expressions in equations (A5) and (A6) where the weights are given by \( 1 - \delta \) (share of workers that sorts on ability) and \( \delta \) (share of workers that sort on preferences) gives us the geometric average of quality of all workers in an occupation in equation (5) where \( \bar{\Gamma} \) is defined as:

\[ \bar{\Gamma} \equiv \Gamma^\delta \tilde{\Gamma}^{1-\delta} \]  

(A7)

**Proof of Proposition 3. Occupational Wage Gaps**

The proof of this proposition is straightforward after substituting the results from propositions 1 and 2 into the expression for the geometric average of the two groups of workers (workers that sort on ability and those that sort on preferences). The geometric average of the wage of all workers is then the geometric mean of the geometric average wage of the two groups of workers.

**Proof of Proposition 4. Relative Propensities**

The proof of this proposition is straightforward after substituting the results from propositions 1 and 3 into the expression for relative propensities \( \frac{p_{ig}}{p_{i,wm}} \).

**Proof of Proposition 5. Relative Labor Force Participation**

This proposition is an application of propositions 3 and 4 to the home sector, assuming no distortions for white men in all sectors and no distortions in the home sector for all groups.

**B Identification and Estimation**

This section explains how we identify and estimate the frictions and other parameters, carried out in the program EstimateTauZ.m.
B.1 Key Equations

To estimate the model, we add one additional feature to the model. In our base case, we assume the return to experience is the same for all occupations, groups, and cohorts. In our robustness checks, however, these parameters may be allowed to vary. We thus index $\gamma$ (and the sum of the experience terms $\bar{\gamma}$) by group $g$ and occupation $i$ in the equations that follow.

The key equations underlying our estimation are listed below.

- **Occupational Choice**

  $$p_i = \frac{\tilde{w}_i^\theta}{\sum_s \tilde{w}_s^\theta}$$

  where $\tilde{w}_{ig} = \frac{w_i \bar{h}_ig \bar{\gamma}_{ig} \phi_i \left[ (1 - s_i) z_{ig} \right]^{1-\beta}}{\tau_{ig}}$

  and $\tau_{ig} \equiv \frac{(1 + \tau^h_{ig}) \eta}{1 - \tau^w_{ig}}$

- **Average Quality (geometric mean)**

  $$e^{E \log (h_{ig}(c,t) \epsilon_{ig}(c))} = \Gamma \phi_i(t) \gamma_{ig}(t-c) \left[ \eta \frac{1 - \tau^w_{ig}(c)}{1 + \tau^h_{ig}(c)} w_i(c) \bar{h}_ig \bar{\gamma}_{ig} s_i(c) \phi(c) \right]^{\frac{\eta}{1-\delta}} \left( \frac{1}{p_{ig}(c)} \right)^{\frac{1}{\eta(1-\delta)}}$$

- **Average Wage (geometric mean)**

  $$\text{wage}_{ig}(c,t) \equiv (1 - \tau^w_{ig}(t)) w_i(t) \gamma_{ig}(t-c) e^{E \log (h_{ig}(c,t) \epsilon_{ig}(c))}$$

  $$= \Gamma \bar{\eta} [p_{ig}(c)]^\delta m_g(c) \left[ \frac{1}{\eta(1-\delta)} \left( 1 - s_i(c) \right) z_{ig}(c) \right]^{-\frac{1}{\beta}} \left( \frac{1 - \tau^w_{ig}(t)}{1 - \tau^w_{ig}(c)} \right) \frac{w_i(t) \gamma_{ig}(t-c) s_i(c) \phi(t)}{w_i(c) \bar{h}_ig \bar{\gamma}_{ig} s_i(c) \phi(c)}$$

  where $m_g(c) = \sum_{i=1}^M \tilde{w}_{ig}(c)^\theta$

- **Relative Propensity. If $\delta \neq 1$:**

  $$\frac{p_{ig}(c,c)}{p_{i,wm}(c,c)} = \left( \frac{\bar{h}_{ig}}{h_{i,wm}} \right)^{\frac{\theta}{1-\delta}} \left( \frac{\tau_{ig}(c,c)}{\tau_{i,wm}(c,c)} \right)^{-\frac{\theta}{1-\delta}} \left( \frac{\text{wage}_{ig}(c,c)}{\text{wage}_{i,wm}(c,c)} \right)^{\frac{\theta(1-\eta)}{1-\delta}} \left( \frac{\bar{\gamma}_{ig}}{\bar{\gamma}_{i,wm}} \right)^{\frac{\theta \eta}{1-\delta}}$$
or when $\delta = 1$, the relative propensity equation simplifies to

$$\frac{\text{wage}_{ig}(c,c)}{\text{wage}_{i,wm}(c,c)} = \left[ \frac{\bar{h}_{ig}/\bar{h}_{i,wm}}{\tau_{ig}(c,c)/\tau_{i,wm}(c,c)} \cdot \left( \frac{\tau_{ig}(c,c)}{\tau_{i,wm}(c,c)} \right)^{\eta} \right]^{1/(1-\eta)}$$

### B.2 Estimating $\theta$, $\eta$, and $\beta$

Given our assumptions, wages within an occupation for a given group follow a Fréchet distribution with the shape parameter $\theta (1 - \eta)$. This reflects both comparative advantage (governed by $1/\theta$) and amplification from endogenous human capital accumulation (governed by $1/(1 - \eta)$). Using micro data from the U.S. Population Census/ACS, we estimate $\theta (1 - \eta)$ to fit the distribution of the residuals from a cross-sectional regression of log hourly wages on 66x4x3 occupation-group-age dummies in each year. We use MLE, with the likelihood function taking into account the number of observations which are top-coded in each year. The resulting estimates for $\theta (1 - \eta)$ range from a low of 1.24 in 1980 to a high of 1.42 in 2000, and average 1.36 across years.\(^{37}\)

The parameter $\eta$ denotes the elasticity of human capital with respect to education spending and is equal to the fraction of output spent on human capital accumulation. Spending on education (public plus private) as a share of GDP in the U.S. averaged 6.6 percent over the years 1995, 2000, 2005, and 2010.\(^{38}\) Since the labor share in the U.S. in the same four years was 0.641, this implies an $\eta$ of 0.103.\(^{39}\) With our base estimate of $\theta (1 - \eta) = 1.36$, $\eta = 0.103$ gives us $\theta = 1.52$.

We can also estimate $\theta$ from the elasticity of labor supply. In our model, the extensive margin elasticity of labor supply with respect to a wage change is $\theta (1 - LFP_g)$. The meta analysis in Chetty et al. (2012) suggests an extensive margin labor supply elasticity of about 0.26 for men. The underlying data in their meta analysis come from the 1970-2007 period. In 1990, roughly in the middle of their analysis, 89.9 percent of men aged 25–34 were in the labor force. To match a labor supply elasticity of 0.26, our model implies that $\theta$ would equal 2.57. This is higher than the estimate of $\theta$ we get from wage dispersion. As a compromise between our two estimates, we will use $\theta = 2$ as our base

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\(^{37}\)Sampling error is minimal because there are 300-400k observations per year for 1960 and 1970 and 2-3 million observations per year from 1980 onward. We did not use 2010 data because top-coded wage thresholds differed by state in that year.


\(^{39}\)Labor share data are from https://research.stlouisfed.org/fred2/series/LABSHPUSA156NRUG. The young's share of earnings is from the U.S. Population Census/ACS.
case, but will also provide results with $\theta$ of 1.5 and 4.

Finally, $\beta$ is the geometric weight on consumption relative to time in an individual’s utility function (1). This parameter is needed to help distinguish preferences from labor market and human capital frictions. As schooling trades off time for consumption, wages must increase more steeply with schooling when people value time more (i.e. when $\beta$ is lower). We choose $\beta = 0.231$ to match the Mincerian return to schooling across occupations, which averages 12.7% across the six decades.\(^{40}\)

B.3 Estimate $\phi_i, z_{i,wm},$ and $w_i$ from data of young white men

The following refers to the program `solveWMfor_wZ.m`. This program uses data on wages, years of schooling, and occupational shares of young white men to estimate $w_i, z_{i,wm}$ and $\phi_i$.

The $s_i$ and $\phi_i$ are determined in a straightforward fashion from years of schooling for young white men in each cohort. In particular, we assume that the pre-market period is 25 years long so that

$$s_i = \frac{\text{Years of Education}}{25}$$

Then $\phi_i$ is determined by the individuals first-order condition for schooling. Rearranging equation (3) gives

$$\phi_i = \frac{1 - \eta}{3\beta} \cdot \frac{s_i}{1 - s_i}.$$ 

Next, we have 67 values of $z_{i,wm}$ as well as $m_{wm}$ to recover, for a total of 68 parameters. However, we only observe wages in 66 occupations for young white men (there is no wage data for the home occupation), so we need two further assumptions to pin down these parameters. One assumption is that $z_{\text{home,wm}} = 1$. The other is that average earnings per person in the home sector for young white men are equal to their average earnings in some other occupation. We choose “Secretaries” for this other occupation, but the results are robust to choosing another occupation (such as Sales).

\(^{40}\)We find the Mincerian return across occupations $\psi$ from a regression of log average wages on average schooling across occupation-groups, with group dummies as controls. The average wage of group $g$ in occupation $i$ is proportional to $(1 - s_i)\psi_g$. We let $s$ be years of schooling divided by a pre-work time endowment of 25 years. Thus the Mincerian return $\psi +/1 - 1$ year around mean schooling $s$ should satisfy $e^{2\psi} = \left(\frac{1 + 0.04}{1 - 0.04}\right)^{\frac{s}{25}}$. The implied $\beta = \ln \left(\frac{1 + 0.04}{1 - 0.04}\right) / (6\psi)$. We set $\beta = 0.231$, the average of the implied $\beta$ values across years. This method allows the model to approximate the Mincerian return to schooling across occupations.
Then, we use the equation for the average wage to back out $m_{wm}$ from the home occupation, since $z_{home} = 1$. After omitting the indices for cohort and time, the specific equation is:

$$m_{wm} = \left[ \text{wage}_{i,wm} \left( 1 - s_i \right) z_{i,wm} \right]^{\frac{1}{\eta}} \cdot \frac{\pi_i}{\gamma_{i,wm}} \cdot \frac{(1 - \eta)}{\theta \cdot p_{ig}(c) - \delta}$$

where $i = home$. Furthermore, we need to make an initial guess about the return to experience term $\gamma$ (We describe later how we do this).

Third, we estimate $z_{i,wm}$ for the other occupations from the equation we use above to back out $m_{wm}$ from data on wages. In this case, we use data on the average wage in the occupation, and the estimate for $m_{wm}$ we obtained from step 2 to back out the $z_{i,wm}$ that fits the wage equation.

Fourth, we estimate $w_i$ from the observed occupational shares. After some algebra, the occupational share equation can be expressed as:

$$w_i = \frac{[p_{i,wm} \cdot m_{wm}]^{\frac{1}{\eta}}}{\pi_i \cdot s_i^{\phi_i} \left( 1 - s_i \right) z_{i,wm}}$$

Again, $\tau = 1$ for white men so these two terms do not show up.

Fifth, we estimate $\gamma$ and $\bar{\gamma}$ (remember we assumed a value for the experience terms for the previous steps) from the change in the average wage of a given cohort and occupation over time. Specifically, the ratio of the average wage in an occupation at time $t$ to that at time $c$ is:

$$\frac{\text{wage}_{i,wm}(c, t)}{\text{wage}_{i,wm}(c, c)} = \frac{w_i(t)(\gamma_{ig}(t - c)) s_i^{\phi(t)}}{w_i(c) s_i^{\phi(c)}}$$

We estimate $\gamma_{i,wm}(t - c)$ from the change in the average wage in an occupation, after controlling for the change in $w_i$ and the returns to schooling. In our base case, we assume $\gamma_{i,wm}(t - c)$ is the same across all occupations and cohorts so simply take the average across all occupations and cohorts.
B.4 Estimating $\tau$

The next part of the estimation obtains the composite of the distortions $\tau_{ig} \equiv \frac{(1+\tau^h)^\eta}{1-\tau^w}$. Remember we assume $\tau^w_{i,wm} = \tau^h_{i,wm} = 0$ and $\bar{h}_{ig} = \bar{h}_{i,wm}$. These two normalizations imply that we can express relative propensities as:

$$\tau_{ig} = \hat{p}_{ig}^{1-\alpha} \cdot \hat{\text{wage}}_{ig}^{(1-\eta)} \cdot \hat{\gamma}_{ig}$$

where a “hat” denotes the value of the variable relative to white men. In this equation, $\hat{\text{wage}}_{ig}$ and $\hat{p}_{ig}$ are data and $\hat{\gamma}_{ig}$ and $\hat{\gamma}_{ig}$ are estimated from the previous step.

B.5 Estimating $\tau^w$, $\tau^h$, and $z$

The next step is to estimate $z$ and the components of $\tau$ (i.e. $\tau^w$ and $\tau^h$) for the other groups (non-white men). This is done in the program estimatetauz.m. We define $\alpha$ as the Cobb-Douglas split of $\tau$ that recovers $1 - \tau^w$. Specifically,

$$\tau^\alpha = \frac{1}{1 - \tau^w} \text{ and } \tau^{1-\alpha} = (1 + \tau^h)^\eta$$

This implies the following definitions of $\tau^w$ and $\tau^h$ as a function of $\tau$ and $\alpha$:

$$\tau^w = 1 - \tau^{-\alpha}$$
$$\tau^h = (\tau^{1-\alpha})^{\frac{1}{\eta}} - 1$$

Our estimation of $\tau^w$ and $\tau^h$ is expressed in terms of $\alpha$.

First, the home sector is assumed to be undistorted, so $\tau^w$ and $\tau^h$ for that sector are set to zero. We then use the "relative propensity" key equation for $\hat{p}_{ig}$ at the start of this section, together with the wage in the home sector for white men, to recover the wage at home for the other groups.

Second, we normalize $z = 1$ for the home sector and back out $m_g$ for the group based on data on the average wage in the home sector. Specifically, after some manipulation, the average wage equation for the sector can be expressed as:

$$m_g(c) = \left[ \frac{\text{wage}_{\text{home},g}(c,c)(1-s_{\text{home}}(c))}{\Gamma \bar{\eta}} \right] \frac{\theta(1-\eta)}{\hat{\gamma}_{\text{home},g}} \cdot \hat{p}_{\text{home},g}(c)^{-\delta}$$
For the other sectors, we use the same wage equation to back out \( z \). Specifically, the wage equation can be expressed as:

\[
z_{ig} = \frac{1}{1 - s_i} \cdot \left[ \bar{\Gamma} \bar{\eta} \left( p_{lg} m_{lg} \right)^{n(1-\eta)} \gamma_{ig} \frac{1}{\tau_{ig}} \frac{1}{\text{wage}_{ig}} \right]^{3/\beta}
\]

We now have \( z \) for all cohorts and \( \tau^w \) and \( \tau^h \) for the young cohort in 1960. What is left is to pin down \( \tau^w \) and \( \tau^h \) for the years after 1960. From the “Average Wage” equation in our list of key equations, we can express wage growth in a given group-occupation as

\[
\frac{\text{wage}_{ig}(c, t+1)}{\text{wage}_{ig}(c, t)} = \frac{1 - \tau^w_{ig}(t+1)}{1 - \tau^w_{ig}(t)} \cdot \frac{w_i(t + 1)}{w_i(t)} \cdot \frac{\gamma_{ig}(t + 1 - c)}{\gamma_{ig}(t - c)} \cdot \frac{s_i(c)^{\phi_i(t+1)}}{s_i(c)^{\phi_i(t)}}
\]

We use solve this equation for \( \tau^w_{ig}(c, t + 1) \) and this becomes our estimate since everything else in the equation is now observed. Then, \( \tau^h_{ig}(t + 1) \) is obtained from \( \tau_{ig}(t + 1) \). In other words, \( \tau^w \) is the time effect in wage growth, while \( \tau^h \) is the cohort effect.

There are two small modifications we make to this in practice. First, we set the minimum value of \( \tau^h \) to \(-0.80\), though we relax this constraint in the robustness checks (without this constraint, the revenue required to subsidize women secretaries with \( \tau^h \) gets implausibly large).

Second, in our model, occupations are chosen when young, so all groups have the same labor-force participation when middle-aged and old. In the data, this is clearly not the case. Therefore, we strip out from wage growth for a given group-occupation using our model’s estimate of the selection effect from differential participation. Based on the “Relative Propensity” equation in our “Key Equation” list, this effect has an elasticity of \( \theta(1 - \eta)/(1 - \delta) \). Absent data on labor-force participation by group, we use a common adjustment across all occupations to obtain the wage growth estimate used in equation (B1):

\[
\left( \frac{\text{wagegrowth}_{ig}}{\text{wagegrowth}_{i,wm}} \right)^{\text{for estimation}} = \left( \frac{\text{wagegrowth}_{ig}}{\text{wagegrowth}_{i,wm}} \right)^{\text{data}} \cdot \left( \frac{\text{LFPgrowth}_{ig}}{\text{LFPgrowth}_{i,wm}} \right)^{\frac{1-\delta}{n(1-\eta)}}
\]

(B2)

We also report results without making this adjustment in our robustness checks.
B.6 Geometric and Arithmetic Averages

To get a closed-form solution, our model relies on geometric averaging, both with the taste and ability types and in particular across the two types. In the micro data, however, we cannot distinguish the two types and take a simple arithmetic average. This section describes how we go from the arithmetic average in the data to the geometric average (of the wage or the quality of workers) in the model. In particular, the formulas below apply specifically to the average wage, but a similar argument applies to average quality.

To see how these are related, let $x \sim \text{Frechet}(\alpha)$ with $\mu_x \equiv \mathbb{E}x = S^\alpha \Gamma(1 - 1/\alpha)$ where $\Gamma(\cdot)$ is the gamma function. It is straightforward to show that $\log x \sim \text{Gumbel}(1/\alpha)$ with mean $\mathbb{E}\left[\log x\right] = \log S + \gamma_{em}/\theta$, where $\gamma_{em} \approx 0.5772$ is the Euler-Macheroni constant. Finally, if $g_x \equiv e^{\mathbb{E}\log x}$ denotes the geometric mean of $x$, then the arithmetic mean and the geometric mean are related by a constant factor of proportionality:

$$\mu_x = g_x \cdot \frac{\Gamma(1 - 1/\alpha)}{e^{\gamma_{em}/\alpha}}. \quad (\text{B3})$$

Now let $G$ denote a geometric average (e.g. of the wage) and let $A$ denote an arithmetic average. In the model, we have

$$G = G_a^{1-\delta} C_t^\delta$$

where $G_a$ is the geometric mean within the ability types and $G_t$ is the geometric mean within the taste types. As part of the proof of Proposition 3, we showed that

$$G_a = \tilde{\Gamma} S$$

$$G_t = \Gamma S p_{ig}^{\gamma(1-\eta)}$$

where $S$ denotes most of the “stuff” in the model. This means that

$$G = G_a^{1-\delta} C_t^\delta = \tilde{\Gamma} S p_{ig}^{\gamma(1-\eta) \delta}. \quad (\text{B4})$$

Because heterogeneity in the ability types is Frechet, by equation (B3) above, the
arithmetic means are given by
\[ A_a = G_a \frac{\Gamma}{\tilde{\Gamma}} \]
and, because there is no heterogeneity among the taste types
\[ A_t = G_t. \]

Then the arithmetic mean in the data is related to the geometric means by
\[
A = (1 - \delta)A_a + \delta A_t \\
= (1 - \delta)G_a \frac{\Gamma}{\tilde{\Gamma}} + \delta G_t \\
= (1 - \delta)\Gamma S + \delta \Gamma S p_{i9}^{\frac{1}{1-\eta}} \\
= \Gamma S \left( 1 - \delta + \delta p_{i9}^{\frac{1}{1-\eta}} \right)
\]
Combining this last result with (B4) gives the key relationship between the arithmetic mean of the wage or quality and the geometric mean:

\[
G = A \cdot \frac{\Gamma}{\tilde{\Gamma}} \cdot \frac{\delta p_{i9}^{\frac{1}{1-\eta}}}{1 - \delta + \delta p_{i9}^{\frac{1}{1-\eta}}} \quad (B5)
\]
Given the arithmetic mean from the data, this is how we construct the geometric mean for the wage or quality that is used in the estimation of the model.

C Numerically Solving for an Equilibrium

The numerical solution of the equilibrium of the model begins by guessing values for \( Y(t) \) and \( m_g(c) \), where \( c = 7 - t \) is the cohort born at date \( t \). Given these values, we compute the equilibrium solution for year \( t \). The main part of this solution is solving for the wages per unit of quality \( w_i(t) \) in each occupation. These are chosen to clear the labor market in each occupation, as in Proposition 6.

The only subtlety in this process is that Proposition 2 characterizes the geometric average of quality in each occupation, while the equilibrium depends on the arithmetic
average instead.41

Perhaps not surprisingly, it is straightforward to show that the arithmetic mean of quality in each occupation, corresponding to Proposition 2, is

$$\Gamma \left( 1 - \frac{1}{\theta(1 - \eta)} \right) S \left[ (1 - \delta) \left( \frac{1}{p_{ig}} \right) \frac{1}{\eta(1 - \eta)} + \delta \right]$$

where $S \equiv s_i(c)\varphi_i(t)\gamma(t - c) \left( \eta s_i(c)\varphi_i(1 - \tau w_i(c)h_i) \bar{\gamma} \right)^{\frac{1}{1 - \eta}}$. This is the expression we use in determining the supply of talent in each occupation when solving for the equilibrium.

D Further Model Implications

While our model is stylized in many respects, it is able to match at least two other important facts that were not targeted in the estimation: trends in female labor supply elasticities and cross-state variation in survey measures of racial discrimination. In this section of the Appendix, we discuss these results.

D.1 Trends in Female Labor Supply Elasticities

Using data from the Current Population Survey, Blau and Kahn (2007) estimate that there was a dramatic decline in female labor supply elasticity during the 1980–2000 period. Helpful for comparing with the predictions of our model, they report female labor supply elasticities specifically for 25-34 year olds. We compare the model’s implied labor supply elasticities — equal to $\theta(1 - LPF_g)$ — for young white women to the estimated labor supply elasticities reported in Blau and Kahn (2007). Using our baseline $\theta$, the model matches both the level and the trend female labor supply elasticities well. Blau and Kahn (2007) report labor supply elasticities for women aged 25-34 of 0.75, 0.60 and 0.35, respectively in 1980, 1990, and 2000 — a change of 0.40 over the time.

41 To see how these are related, let $x \sim \text{Frechet}(\theta)$ with $\mu_x \equiv E[x] = S \Gamma(1 - 1/\theta)$ where $\Gamma(\cdot)$ is the gamma function. It is straightforward to show that $\log x \sim \text{Gumbel}(1/\theta)$ with mean $E[\log x] = \log S + \gamma_{em}/\theta$, where $\gamma_{em} \approx 0.5772$ is the Euler-Machceroni constant. Finally, if $g_x \equiv e^{\log x}$ denotes the geometric mean of $x$, then the arithmetic mean and the geometric mean are related by a constant factor of proportionality:

$$\mu_x = g_x \cdot \frac{\Gamma(1 - 1/\theta)}{e^{\gamma_{em}/\theta}}.$
period. Our comparable model estimates for young women are 0.90, 0.70, and 0.65 for the three years - a change of 0.25 over the time period. Our estimates are only slightly higher in levels than the Blau and Khan estimates over the three years with a roughly similar trend.

Nothing in our model is calibrated to match either the level or the trend in labor supply elasticities for women. As discussed earlier, we estimated $\theta$ to match the labor supply elasticity of men in 1980. With that parameter pinned down, our model implies that women's labor supply elasticity is only a function of female labor force participation. The fact that we can roughly match the level of the labor supply elasticity for young women in three different time periods suggests that our model is consistent with empirical moments outside the ones we used to calibrate the model.

D.2 Cross State Measures of Discrimination

There are very few micro-based measures of discrimination to which we can compare our estimated $\tau$'s. One such exception is the recent work by Charles and Guryan (2008). Charles and Guryan (CG) used data from the General Social Survey (GSS) to construct a measure of the taste for discrimination against blacks for every state. The GSS asks a large nationally representative sample of individuals about their views on a variety of issues. A series of questions have been asked over the years assessing the respondents attitudes towards race. For example, questions were asked about individuals' views on cross-race marriage, school segregation, and the ability for homeowners to discriminate with respect to home sales. Pooling together survey questions from the mid 1970s through the early 1990s and focusing only a sample of white respondents, Charles and Guryan make indices of the extent of racial discrimination in each state.\footnote{We focus on their marginal discrimination measure. The concept of the marginal discriminator comes from Becker's theory of discrimination. If there are 10 percent of blacks in the state labor market, it is only the discrimination preferences of the white person at the 10th percentile of the white distribution that matters for outcomes (with the first percentile being the least discriminatory).} Higher values of the CG discrimination measure imply more discrimination. They compute their measure for 44 states.

Figure 12 shows a simple scatter plot between the CG measure of discrimination and our measure $\tau_{bm}$ at the state level.\footnote{From our earlier estimates, we compute a composite $\tau$ measure for black men relative to white men in each U.S. state. To ensure we have enough observations in each state, we make a few simplifying assumptions.} Each observation in the scatter plot is a U.S.
state where the size of the circle represents the number of black men within our Census sample. We also show the weighted OLS regression line on the figure. As seen from the figure, there is a very strong relationship between our measures of $\tau_{bm}$ and the CG discrimination index. The adjusted R-squared of the simple scatter plot is 0.6 and the slope of the regression line is 0.45 with a standard error of 0.06. Places we identify as having a high $\tau_{bm}$ are the same places Charles and Guryan find as being highly discriminatory based on survey data from the GSS. The findings in Figure 12 provide additional external validity that our procedure is measuring salient features of the U.S. economy over the last five decades.

E Appendix Tables and Figures

assumptions. First, we assume that there are no cohort effects in our composite measure of $\tau$. This allows us to pool together all cohorts within a year when computing our measure of $\tau$. Next, we collapse our 67 occupations to 20 occupations; see Appendix Table E2. Also, we pool together data from 1980 and 1990; we do this because the CG discrimination measure is based on data pooled from the GSS between 1977 and 1993. We then aggregate $\tau_{i,bm}$ from our 20 different occupations to one measure of $\tau_{bm}$ for each state by taking a weighted average of the occupation level $\tau$ where the weights are based on share of the occupations income (for the country as whole) out of total income across all occupations (for the country as a whole). Finally, we exclude states with an insufficient number of black households to compute our measure of $\tau_{bm}$. Given the CG restrictions from the GSS and our restrictions from the Census data, we are left with 37 states.
Figure 12: Model τ’s for Black Men vs. Survey Measures of Discrimination, by U.S. State

Note: Figure plots measures of our model’s implied composite τ’s for black men for each state using pooled data from the 1980 and 1990 census (x-axis) against survey-based measures of discrimination against blacks for each state as reported in Charles and Guryan (2008). The Charles and Guryan data are compiled using data from the General Social Survey between 1977 and 1993. We use their marginal discrimination measure for this figure. See text for additional details.
Table E1: Sample Statistics by Census Year

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
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<tbody>
<tr>
<td>Sample Size</td>
<td>624,579</td>
<td>674,059</td>
<td>3,943,034</td>
<td>4,607,829</td>
<td>5,084,891</td>
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<td>Share of Sample:</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>White Men, Age 25-34</td>
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<td>0.185</td>
<td>0.172</td>
<td>0.132</td>
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<td>White Men, Age 35-44</td>
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<td>0.131</td>
<td>0.156</td>
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<tr>
<td>White Men, Age 45-55</td>
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<td>0.118</td>
<td>0.108</td>
<td>0.139</td>
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<td>0.175</td>
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<td>White Women, Age 35-44</td>
<td>0.171</td>
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<td>0.159</td>
<td>0.163</td>
<td>0.137</td>
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<tr>
<td>White Women, Age 45-55</td>
<td>0.145</td>
<td>0.157</td>
<td>0.127</td>
<td>0.113</td>
<td>0.143</td>
<td>0.162</td>
</tr>
<tr>
<td>Black Men, Age 25-34</td>
<td>0.015</td>
<td>0.016</td>
<td>0.021</td>
<td>0.023</td>
<td>0.021</td>
<td>0.022</td>
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<tr>
<td>Black Men, Age 35-44</td>
<td>0.015</td>
<td>0.015</td>
<td>0.014</td>
<td>0.019</td>
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<td>0.022</td>
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<tr>
<td>Black Men, Age 45-55</td>
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<td>0.014</td>
<td>0.012</td>
<td>0.012</td>
<td>0.017</td>
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<tr>
<td>Black Women, Age 25-34</td>
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<td>Black Women, Age 35-44</td>
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<tr>
<td>Black Women, Age 45-55</td>
<td>0.015</td>
<td>0.016</td>
<td>0.015</td>
<td>0.015</td>
<td>0.020</td>
<td>0.026</td>
</tr>
</tbody>
</table>

Note: Data comes from the 1960-2000 U.S. Censuses and the pooled 2010 American Community Survey (ACS). Samples restricted to black and white, men and women between the ages of 25 and 54. Those in the military are excluded. Also, excluded are those not working but actively searching for a job. Sample shares are weighted using Census and ACS provided sample weights.
### Table E2: Occupation Categories for our Base Occupational Specifications

<table>
<thead>
<tr>
<th></th>
<th>Occupational Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.</td>
<td>Home Sector (0)</td>
</tr>
<tr>
<td>1.</td>
<td>Executives, Administrative, and Managerial (1)</td>
</tr>
<tr>
<td>2.</td>
<td>Management Related (2)</td>
</tr>
<tr>
<td>3.</td>
<td>Architects (3)</td>
</tr>
<tr>
<td>4.</td>
<td>Engineers (3)</td>
</tr>
<tr>
<td>5.</td>
<td>Math and Computer Science (3)</td>
</tr>
<tr>
<td>6.</td>
<td>Natural Science (4)</td>
</tr>
<tr>
<td>7.</td>
<td>Health Diagnosing (5)</td>
</tr>
<tr>
<td>8.</td>
<td>Health Assessment (6)</td>
</tr>
<tr>
<td>9.</td>
<td>Therapists (6)</td>
</tr>
<tr>
<td>10.</td>
<td>Teachers, Postsecondary (7)</td>
</tr>
<tr>
<td>11.</td>
<td>Teachers, Non-Postsecondary (8)</td>
</tr>
<tr>
<td>12.</td>
<td>Librarians and Curators (8)</td>
</tr>
<tr>
<td>13.</td>
<td>Social Scientists and Urban Planners (4)</td>
</tr>
<tr>
<td>14.</td>
<td>Social, Recreation, and Religious Workers (4)</td>
</tr>
<tr>
<td>15.</td>
<td>Lawyers and Judges (5)</td>
</tr>
<tr>
<td>16.</td>
<td>Arts and Athletes (4)</td>
</tr>
<tr>
<td>17.</td>
<td>Health Technicians (9)</td>
</tr>
<tr>
<td>18.</td>
<td>Engineering Technicians (9)</td>
</tr>
<tr>
<td>19.</td>
<td>Science Technicians (9)</td>
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<td>24.</td>
<td>Records Processing, Non-Financial (11)</td>
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<td>25.</td>
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<td>26.</td>
<td>Office Machine Operator (11)</td>
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<td>28.</td>
<td>Mail Distribution (11)</td>
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<tr>
<td>29.</td>
<td>Scheduling and Distributing Clerks (11)</td>
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<td>30.</td>
<td>Adjusters and Investigators (11)</td>
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<td>31.</td>
<td>Misc. Administrative Support (11)</td>
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<td>Private Household Occupations (13)</td>
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<td>Precision Textile (16)</td>
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<td>53.</td>
<td>Precision Other (16)</td>
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<td>Woodworking Machine Operator (17)</td>
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<td>Printing Machine Operator (17)</td>
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<td>Machine Operator, Other (19)</td>
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<td>Freight, Stock, &amp; Material Handlers (18)</td>
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Notes: Our 66 market occupations are based on the 1990 Census Occupational Classification System. We use the 66 sub-headings (shown in the table) to form our occupational classification. See http://www.bls.gov/nls/quex/r1/y97r1cbka1.pdf for the sub-heading as well as detailed occupations that correspond to each sub-heading. As discussed in the text, we include the home sector as an additional occupation. When computing racial barriers at the state level, we use only twenty broader occupations. The number in parentheses refers to how we group these 67 occupations into the twenty broader occupations for the cross-state analysis. For example, all occupations with a 11 in parentheses refers to the fact that these occupations were combined to make the 11th occupation in our broader occupation classification.