EXTERNALITIES AND GROWTH

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Abstract

Externalities play a central role in most theories of economic growth. We argue that international externalities, in particular, are essential for explaining a number of empirical regularities about growth and development. Foremost among these is that many countries appear to share a common long run growth rate despite persistently different rates of investment in physical capital, human capital, and research. With this motivation, we construct a hybrid of some prominent growth models that have international knowledge externalities. When calibrated, the hybrid model does a surprisingly good job of generating realistic dispersion of income levels with modest barriers to technology adoption. Human capital and physical capital contribute to income differences both directly (as usual), and indirectly by boosting resources devoted to technology adoption. The model implies that most of income above subsistence is made possible by international diffusion of knowledge.

Keywords

externalities, spillovers, growth, technology, diffusion

JEL classification: O11, O33, O40

If ideas are the engine of growth and if an excess of social over private returns is an essential feature of the production of ideas, then we want to go out of our way to introduce external effects into growth theory, not try to do without them.

Robert E. Lucas (2002, p. 6)

1. Introduction

A number of facts suggest that international knowledge externalities are critical for understanding growth and development. The growth slowdown that began in the early 1970s was world-wide, not an OECD-only phenomenon. Countries with high investment rates exhibit higher income levels more than higher growth rates. Country growth rate differences are not very persistent from decade to decade, whereas differences in country incomes and investment rates are highly persistent. These patterns hold for investment rates in physical, human, and research capital. Together, they suggest that investment rates affect country transitional growth rates and long run relative incomes rather than long run growth rates. They also suggest countries are subject to the same long run growth rate. We argue that this represents evidence of very large international spillovers at the heart of the long run growth process.

We organize this chapter as follows. In Section 2 we describe two broad types of externalities and the growth models that do (and do not) feature them. Section 3 presents cross-country evidence that, we argue, is very hard to reconcile with the models that have no international externalities. Section 4 calibrates a model of growth with international externalities in the form of technology diffusion. The implied externalities are huge. Section 5 concludes and points out directions for future research.

2. A brief guide to externalities in growth models

In this section we briefly discuss the role that externalities play in prominent theories of economic growth. One class of growth theories features externalities in the accumulation of knowledge possessed by firms (organizational capital) or by workers (human capital). Another class of growth models features externalities from the introduction of new goods, in the form of surplus to consumers and/or firms. Other theories combine knowledge externalities and new good externalities. Finally, some important growth theories include no externalities at all. Table 1 provides examples of growth models categorized in these four ways. At the end of this section, we will dwell a little on the predictions of no-externalities models in order to motivate the evidence we describe in the next section. The evidence in the next section will suggest that models with no externalities cannot explain a number of empirical patterns.

	New good externalities	No new good externalities
Knowledge externalities	Stokey (1988, 1991)	Romer (1986)
	Romer (1990)	Lucas (1988, 2004) Tamura
	Aghion and Howitt (1992)	(1991)
	Eaton and Kortum (1996)	Parente and Prescott (1994)
	Howitt (1999, 2000)	
No knowledge externalities	Rivera-Batiz and Romer	Jones and Manuelli (1990)
	(1991) Romer (1994)	Rebelo (1991) Acemoglu
	Kortum (1997)	and Ventura (2002)

Table 1 Some growth models by type of externality

2.1. Models with knowledge externalities

Romer (1986) modeled endogenous growth due to knowledge externalities: a given firm is more productive the higher the average knowledge stock of other firms. As an example, consider a set of atomistic firms, each with knowledge capital k, benefiting from the average stock of knowledge capital in the economy K in their production of output y:

$$y_{it} = Ak_{it}^{\alpha} K_t^{1-\alpha}, \quad 0 < \alpha < 1. \tag{2.1}$$

Romer showed that, under certain conditions, constant returns to economy-wide knowledge, as in this example, can generate endogenous growth. The external effects are, of course, critical for long-run growth given the diminishing returns to private knowledge capital. Romer was agnostic as to whether the knowledge capital should be thought of as disembodied (knowledge in books) or embodied (physical capital and/or human capital).

Lucas (1988) was more specific, stressing the importance of human capital. Lucas sketched two models, one with human capital accumulated off-the-job and another with human capital accumulated on-the-job (i.e., learning by doing). Both models featured externalities. In the model with human capital accumulated off-the-job, Lucas posited

$$y_{it} = Ak_{it}^{\alpha} [u_{it}h_{it}n_{it}]^{1-\alpha}H_t^{\gamma}, \quad \text{with } \gamma > 0, \quad \text{and}$$
(2.2)

$$h_{it+1} = h_{it} + Bh_{it}[1 - u_{it}]$$
 with $0 < u_{it} < 1$. (2.3)

Here u is the fraction of time spent working, and 1-u is the fraction of time spent accumulating human capital; h is an individual worker's human capital, and H is economy-wide average human capital; k and n are physical capital and number of workers at a given firm. Because human capital accumulation is linear in the level of human capital, human capital is an engine of growth in this model. This is true with or without the externalities; across-dynasty externalities are not necessary for growth. As Lucas discusses, however, within-dynasty human capital spillovers are implicit if one imagines (2.3) as successive generations of finite-lived individuals within a dynasty.

Within-dynasty externalities, however, would not have the same normative implications as across-dynasty externalities, namely underinvestment in human capital. Lucas (1988) did not argue that across-dynasty externalities were needed to fit particular facts. But he later observed that such across-household externalities could help explain why we see "immigration at maximal allowable rates and beyond from poor countries to wealthy ones" [Lucas (1990, p. 93)].

Tamura (1991) analyzed a human capital externality in the production of human capital itself. This formulation conformed better to the intuition that individuals *learn* from the knowledge of others. Tamura specified

$$y_{it} = Ak_{it}^{\alpha} [u_{it}h_{it}n_{it}]^{1-\alpha}, \qquad (2.4)$$

$$h_{it+1} = h_{it} + B(h_{it}[1 - u_{it}])^{\beta} H_t^{1-\beta}.$$
 (2.5)

Because H represents economy-wide average human capital, $\beta < 1$ implies that learning externalities are essential for sustaining growth in Tamura's setup. If applied to each country, this model would suggest that immigrants from poor to rich countries should enjoy fast wage growth after they migrate, as they learn from being around higher average human capital in richer countries. Lucas (2004) used such learning externalities within cities as an ingredient of a model of urbanization and development.

Models not always thought of as having knowledge externalities are Mankiw, Romer and Weil's (1992) augmented Solow model and the original Solow (1956) neoclassical growth model. In Solow's model all firms within the economy enjoy the same level of TFP. This common level of TFP reflects technology accessible to all. The Solow model therefore does feature disembodied knowledge externalities across firms within an economy. In Mankiw et al.'s extension, knowledge externalities flow across countries as well as across firms within countries. In Section 4 we will discuss models with more limited international diffusion of knowledge. In these models imperfect diffusion means differences in TFP can play a role in explaining differences in income levels and growth rates. We stress that the Mankiw et al. model relies on even stronger externalities than the typical model of international technology spillovers, such as Parente and Prescott (1994) or Barro and Sala-i-Martin (1995, Chapter 8). We will discuss these models at greater length in Section 4, when we calibrate a hybrid version of them.

2.2. Models with knowledge externalities and new-good externalities

Models with both knowledge externalities and new-good externalities are the most plentiful in the endogenous growth literature. By "new-good externalities" we mean surplus to consumers and/or firms from the introduction of new goods. The new goods take the form of new varieties and/or higher quality versions of existing varieties. In Stokey (1988), learning by doing leads to the introduction of new goods over time. The new goods are of higher quality, and eventually displace older goods. The learning is completely external to firms, and what is learned applies to new goods even more than older goods. Hence learning externalities are at the heart of her growth process. In Stokey

(1991), intergenerational human capital externalities (the young learn from the old) are critical for human capital accumulation. Human capital accumulation, in turn, facilitates the introduction of higher quality goods, which are intensive in human capital in her model.

Quality ladder models – pioneered by Grossman and Helpman (1991, Chapter 4) and Aghion and Howitt (1992, 1998) – feature knowledge spillovers in that each quality innovation is built on the previous leading-edge technology. Such intertemporal knowledge spillovers are also fundamental in models with expanding product variety, such as Romer (1990) and Grossman and Helpman (1991, Chapter 3). In Romer (1990),

$$Y = H_Y^{\alpha} L^{\beta} \int_0^A x(i)^{1-\alpha-\beta} di, \qquad (2.6)$$

$$\dot{A} = BH_AA. \tag{2.7}$$

Intermediate goods, the x(i)'s, are imperfect substitutes in production. This is the Dixit—Stiglitz "love of variety" model. The stock of varieties, or ideas, is A. In (2.7) new ideas are invented using human capital and, critically, the previous stock of ideas. This is the intertemporal knowledge spillover. Jones (1995, 2002) argues that, in contrast to (2.7), there are likely to be diminishing returns to the stock of ideas (an exponent less than 1 on A). He bases this on the fact that the number of research scientists and engineers have grown in the U.S. and other rich countries since 1950, yet the growth rate has not risen, as (2.7) would predict. Intertemporal knowledge spillovers still play a pivotal role in Jones' specification; they are just not as strong as in Romer's (2.7).

More recent models, such as Eaton and Kortum (1996) and Howitt (1999, 2000), continue to emphasize both knowledge externalities and new-good externalities. We will elaborate on these in Section 4 below.

2.3. Models with new-good externalities

It is hard to find a model with new-good externalities but without knowledge externalities. We have identified three papers in the literature featuring such models, but two of the papers also have versions of their models with knowledge externalities.

Rivera-Batiz and Romer (1991) present a variation on Romer's (1990) model, as part of their analysis of the potential growth gains from international integration. In their twist, new intermediate goods are invented using factors in the same proportions as for final goods production in (2.6):

$$\dot{A} = BH^{\alpha}L^{\beta} \int_{0}^{A} x(i)^{1-\alpha-\beta} di.$$
 (2.8)

They call this the "lab equipment model" to underscore the use of equipment in the research lab, just like in the production of final goods. In this formulation, they emphasize, "Access to the designs for all previous goods, and familiarity with the ideas and know-how that they represent, does not aid the creation of new designs" (pp. 536–537). I.e., there are no knowledge externalities, domestic or international. Production of

ideas is not even knowledge-intensive. Ideas are embodied in goods, however, and there is surplus to downstream consumers from their availability. Rivera-Batiz and Romer note that this model allows countries to benefit from ideas developed elsewhere simply by importing the resulting products. Just as important, international trade allows international specialization in research. Countries can specialize in inventing different products, rather than every product being invented everywhere.

In a similar spirit, Romer (1994) considered a model in which knowledge about how to produce different varieties does not flow across countries, but each country can import the varieties that other countries know how to produce. For a small open economy, Romer posited

$$Y_t = A \left(\sum_{j=1}^{M_t} x_{jt}^{\alpha} \right) N_t^{1-\alpha}, \quad 0 < \alpha < 1.$$
 (2.9)

 x_j represents the quantity of imports of the jth variety of intermediate good. Because $\alpha < 1$, intermediate varieties are imperfectly substitutable in production. Firms in the importing country will have higher labor productivity the more import varieties they can access. If exporters cannot perfectly price discriminate and there is perfect competition among domestic final-goods producers, the higher labor productivity (higher Y/N) will benefit domestic workers/consumers. If consumer varieties were imported as well, there would be an additional source of consumer surplus from import varieties. Romer analyzed the impact of import tariffs on the number of varieties M imported in the presence of fixed costs of importing each variety in each country. Although Romer's model is static, growth in the number of varieties over time, say due to domestic population growth or falling barriers to trade, would be a source of growth in productivity and welfare in his model.

Kortum (1997) develops a model in which researchers draw techniques of varying efficiency levels from a Poisson distribution. Kortum does consider spillovers in the form of targeted search. But he also considers the case of blind search, wherein draws are independent of the previous draws. (Kortum fixes the set of goods produced but allows endogenous research into discovering better techniques for producing each good.) In the case of blind search, there are no knowledge spillovers. Growth is sustained solely because of population growth that raises the supply of and demand for researchers. It takes more and more draws to obtain a quality deep enough into the right tail to constitute an improvement. A constant population growth rate sustains a constant flow of quality improvements and, hence, a constant growth rate of income.

2.4. Models with no externalities

The seminal growth models without externalities are the AK models of Jones and Manuelli (1990) and Rebelo (1991). In the next section we will present evidence at odds with such models, so we dwell on their implications here. We consider a version close

to Rebelo's. Final output is a Cobb-Douglas function of physical and human capital:

$$C_t + I_t = Y_t = AK_{Yt}^{\alpha} H_{Yt}^{1-\alpha},$$
 (2.10)

where K_Y and H_Y represent the stocks of physical capital and human capital devoted to producing current output. As shown, current output can be used for either consumption or investment. The accumulation equations for physical and human capital are, respectively,

$$K_{Yt+1} + K_{Ht+1} = K_{t+1} = (1 - \delta_K)K_t + I_t, \tag{2.11}$$

$$H_{Yt+1} + H_{Ht+1} = H_{t+1} = (1 - \delta_H)H_t + BH_{Ht}^{\gamma}K_{Ht}^{1-\gamma}.$$
 (2.12)

 H_H and K_H represent the stocks of human and physical capital, respectively, devoted to accumulating human capital.

We will focus on an equilibrium with a constant fraction of output invested in physical capital ($s_I = I/Y$) and a constant share of human capital deployed in human capital accumulation ($s_H = H_H/H$). We assume that the ratio of marginal products of physical and human capital are equated across the final output and human capital sectors, so that physical capital is devoted to

$$s_K = K_H/K = \frac{s_H(1-\gamma)(1-\alpha)}{s_H(1-\gamma)(1-\alpha) + (1-s_H)\gamma\alpha}.$$
(2.13)

The balanced growth rate is defined as

$$1 + g = Y_{t+1}/Y_t = K_{t+1}/K_t = H_{t+1}/H_t. (2.14)$$

The level of the balanced growth rate is an implicit function of the investment rates and parameter values:

$$(g + \delta_K)^{1-\gamma} (g + \delta_H)^{1-\alpha} = (A(1 - s_K)^{\alpha} (1 - s_H)^{1-\alpha} s_I)^{1-\gamma} (B s_H^{\gamma} s_K^{1-\gamma})^{1-\alpha}.$$
 (2.15)

Provided $\alpha < 1$, human capital is the engine of growth. The growth rate is monotonically increasing in s_I because physical capital is an input into human capital accumulation whereas consumption is not. Related, the growth rate does not monotonically increase with the share of inputs devoted to producing human capital. This is because devoting more resources to producing current output increases the stock of physical capital, which is an input into human capital accumulation and hence growth. When we look at the data in Section 3, however, we will find no country so high an s_H or s_K as to inhibit its growth according to this model.

¹ To reinforce intuition, consider the (unrealistic) case of $\gamma = 0$, wherein new human capital is produced only with physical capital. In this case, growth is not strictly increasing in s_K (the share of capital devoted to human capital production) because some physical capital itself needs to be devoted to its own production.

When $\alpha = 1$ we have a literal Y = AK model, and the growth rate is solely a function of the physical capital investment rate:

$$g + \delta_K = As_I. \tag{2.16}$$

Here there is no point in devoting effort to producing human capital, so $s_H = 0$ and $s_K = 0$.

In the special case $\gamma=1$, human capital is produced solely with human capital. This might be called a *BH* model. Presuming $\alpha<1$ of course, the growth rate is simply

$$g + \delta_H = Bs_H. \tag{2.17}$$

Unlike when $\gamma < 1$, the growth rate here is monotonically increasing in the effort devoted to adding more human capital. Lucas (1988) and many successors focus on this BH model because human capital accumulation is evidently intensive in human capital. Moreover, even AK models such as Jones and Manuelli (1990) construe their K to incorporate both human capital and physical capital. The consensus for diminishing returns to physical capital ($\alpha < 1$) is strong. Constant returns are entertained only for a broad measure of physical and human capital. We stress (2.15), a hybrid of AK and BH models, because this generalization allows us to take into account the combined impact of physical and human capital investment rates on growth when physical capital is an input to human capital accumulation ($\gamma < 1$).

3. Cross-country evidence

In this section we document a number of facts about country growth experiences over the last fifty years. We show that country growth rates appear to depend critically on the growth and income levels of *other* countries, rather than solely on domestic investment rates in physical and human capital. Cross-country externalities are a promising explanation for this interdependence. In brief, here are the main facts we will present:

- The growth slowdown that began in the mid-1970s was a *world-wide* phenomenon. It hit both rich countries and poor countries, and economies on every continent.
- Richer OECD countries grew much more slowly from 1950 to around 1980, despite the fact that richer OECD economies invested at higher rates in physical and human capital.
- Differences in country investment rates are far more persistent than differences in country growth rates.
- Countries with high investment rates tend to have high levels of income more than they tend to have high growth rates.

3.1. The world-wide growth slowdown

As has been widely documented for rich countries, the growth rate of productivity slowed beginning in the early 1970s.² Less widely known is that the slowdown has

² The causes of the slowdown remain largely a mystery. For example, see Fischer (1988).

	Aver	age Y/L g	growth		Average s	I		Average s	Н
	1960–75	5 1975–00	# of countries		1975–00	# of countries		5 1975–00	# of countries
World	2.7%	1.1%	96	15.8%	15.5%	96	7.1%	9.7%	74
OECD	3.4	1.8	23	23.2	22.9	23	11.4	14.3	21
Non-OECD	2.5	0.9	73	13.5	13.2	73	5.4	8.0	53
Africa	2.0	0.5	38	12.3	10.5	38	3.9	6.0	19
Asia	3.2	2.8	17	14.5	19.9	17	6.9	9.9	16
Europe	3.8	1.9	18	24.9	23.1	18	10.7	13.7	16
North America	2.8	0.4	13	14.3	14.5	13	7.5	10.2	13
South America	2.3	-0.1	10	17.3	15.0	10	7.1	9.8	10
1st quartile (poorest)	1.6	0.5	24	9.6	9.9	24	3.1	5.0	19
2nd quartile	2.6	1.4	24	14.8	14.2	24	5.7	8.9	19
3rd quartile	3.5	1.1	24	15.4	16.3	24	7.5	10.3	18
4th quartile (richest)	3.0	1.5	24	23.6	21.9	24	12.3	15.1	18

Table 2
Output growth declined sharply worldwide

Notes: Y/L is GDP per worker. s_I is the physical capital investment rate, and s_H years of schooling attainment (for the 25+ population) divided by 60 years (working life). Data sources: Barro and Lee (2000) and Heston, Summers and Aten (2002).

been a *world-wide* phenomenon, rather than just an OECD-specific event.³ We document this in Table 2. Across 96 countries, the growth rate in PPP GDP per worker fell from 2.7% per year over 1960–1975 to 1.1% per year over 1975–2000. Growth decelerated 1.6 percentage points on average in both the sample of 23 OECD countries and the in the sample of 73 non-OECD countries.⁴ The slowdown hit North and South America the hardest (their growth rates fell 2.4 percentage points) and barely brushed Asia (who slowed down just 0.4 of a percentage point). The slowdown hit all income quartiles of the 96 country sample (based on PPP income per worker in 1975). Although each income quartile grew at least one percentage point slower, the slowdown was not as severe in the poorest half as in the richest half. China's growth rate actually accelerated from 1.8 to 5.1, in the wake of reforms that began in the late 1970s. Chile, which experienced rapid growth in the 1990s, accelerated 2.1 percentage points.

Why does a world-wide growth slowdown suggest international externalities? Couldn't it simply reflect declining investment rates world-wide, as suggested by the AK model in the previous section? Table 2 also shows what average investment rates in

³ An exception is Easterly (2001b).

⁴ OECD countries are based on 1975 membership. There were 24 OECD members in 1975, but the Penn World Tables contain data for unified Germany only back to 1970.

physical and human capital did before and after the mid-1970s. The investment rates in physical capital come from Penn World Table 6.1. As a proxy for the fraction of time devoted to accumulating more human capital, we used years of schooling attainment relative to a 60-year working life. We used data on schooling attainment for the 25 and older population from Barro and Lee (2000). This human capital investment rate, which averages around 7% across countries, reflects the fraction of ages 5 to 65 devoted to schooling as opposed to working. We prefer the attainment of the workforce as opposed to the enrollment rates of the school-age population. The latter should take a long time to affect the workforce and therefore the growth rate.

According to Table 2, the average investment rate in physical capital across all countries was virtually unchanged (15.8% before vs. 15.5% after the slowdown), and the investment rate in human capital actually rose strongly (going from 7.1% to 9.7%). The same pattern applies for the OECD and non-OECD separately, and for all four quartiles of initial income. Thus the growth slowdown cannot be attributed to a world-wide decline in investment rates.

The breadth of the growth slowdown suggests *something* linking country growth rates, and ostensibly something other than investment rates. This is contrary to the predictions of AK models, in which the growth rate of a country depends on domestic investment rates. The world-wide nature of the slowdown suggests that endogenous growth models, more generally, should not be applied to individual countries but rather to a collection of interdependent countries. Knowledge diffusion through trade, migration, and foreign direct investment are likely sources of interdependence.

Three other examples of interdependence are offered by Parente and Prescott (2005). First, growth rates picked up in the 20th century relative to the 19th century for many countries. Second, the time it takes a country to go from \$2000 to \$4000 in per capita income has fallen over the 20th century, suggesting the potential to grow rapidly by adopting technology in use elsewhere. Third and related, they stress that "growth miracles" always occur in countries with incomes well beneath the richest countries, again consistent with adoption of technology from abroad.

Knowledge diffusion, broadly construed, could include imitation of successful institutions and policies in other countries. Kremer, Onatski and Stock (2001) argue that such imitation might explain the empirical transition matrix of the world income distribution. If improving institutions leads only to static gains in efficiency, however, then the barriers to imitation have to be large to explain why the best institutions are not in place everywhere. As we will illustrate in Section 4 below, the required barriers to technology adoption are modest precisely because the benefits accumulate with investments.

3.2. Beta convergence in the OECD

As documented by Baumol (1986) and many others, incomes have generally been converging in the OECD. Barro and Sala-i-Martin (1995) used the term sigma con-

⁵ It also casts doubt on explanations for the growth slowdown that are confined to rich countries.

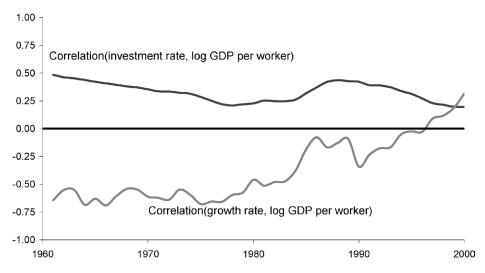


Figure 1. OECD incomes correlate negatively with growth rates, positively with investment rates. Source: Penn World Table 6.1 data on 23 OECD Countries.

vergence to describe such episodes of declining cross-sectional standard deviations in log incomes. We focus on a related concept that Barro and Sala-i-Martin labeled *beta convergence*, namely a negative correlation between a country's initial income level and its subsequent growth rate. We look at beta convergence year by year in Figure 1. The data on PPP income per worker comes from Penn World Table 6.1 [Heston, Summers and Aten (2002)], and covers 23 OECD countries over 1960–2000. The figure shows the correlation between current income and growth hovering between -0.50 and -0.75 from 1960 through the early 1980s. The correlation was still negative from the mid-1980s through the mid-1990s, but less so, and turned positive in the latter 1990s.

DeLong (1988) pointed out that a country's OECD membership is endogenous to its level of income, so that members at time t will tend to converge toward each other's incomes leading up to time t. Our focus, however, is not on convergence per se. Our point is instead about how investment rates correlate with income during the period of convergence. Figure 1 also shows the physical capital investment rate, and it is *positively* correlated with a country's income throughout the sample. Figure 2 shows that schooling attainment is also *positively* correlated with income throughout the sample.

How do these investment correlations square with *AK*-type models with no externalities? Expression (2.15) shows that a country's growth rate should be increasing in its investment rates. For beta convergence to occur in this model, a country's investment rates must be *negatively* correlated with a country's level of income. But Figures 1 and 2 show the opposite is true: in every year, richer OECD countries had *higher* investment rates in human and physical capital than poorer OECD countries did. According to this class of models, OECD countries should have been diverging throughout the entire sample, rather than converging through most of it. Now, this reasoning ignores likely

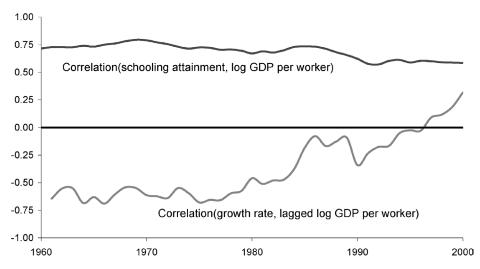


Figure 2. OECD incomes correlate negatively with growth rates, positively with investment schooling. Source: Penn World Table 6.1 and Barro and Lee (2000) data for 21 OECD Countries.

differences in efficiency parameters A and B across countries. But rescuing AK models would require that richer countries have *lower* efficiency parameters. We would guess that rich countries tend to have better rather than worse institutions [e.g., Hall and Jones (1999)].

3.3. Low persistence of growth rate differences

Easterly et al. (1993) documented that country growth rate differences do not persist much from decade to decade. They estimated correlations of around 0.1 to 0.3 across decades. In contrast, they found that country characteristics such as education levels and investment rates exhibit cross-decade correlations in the 0.6 to 0.9 range. Just as we do, they suggest country characteristics may determine relative income levels and world-wide technological changes long-run growth. Easterly and Levine (2001) similarly provide evidence that "growth is not persistent, but factor accumulation is".

In Table 3 we present similar findings. We compare average growth rates from 1980–2000 vs. 1960–1980, and from decade to decade within 1960–2000. We find growth rates much less persistent than investment rates for the world as a whole, and for the OECD and non-OECD separately. Again, these facts seem hard to reconcile with the *AK* model in which a country's domestic investment rates determine its growth rate.

Figure 3 illustrates a related pattern: deciles of countries (based on 1960 income per worker) grew at similar average rates from 1960 to 2000. Each decile consists of the unweighted average of income per worker in 9 or 10 countries. The average growth rate is 1.7% in the sample, and the bottom decile in 1960 grew at precisely this rate. This figure suggests movements in relative incomes, but no permanent differences in

	1980–20	000 vs. 1960–1	980	Decade to decade		
	Y/L growth	s_I	SH	Y/L growth	s_I	s_H
World	0.34	0.56	1.02	0.20	0.77	1.00
	(0.13)	(0.07)	(0.04)	(0.07)	(0.04)	(0.02)
OECD	0.12	0.44	0.86	0.27	0.70	0.92
	(0.13)	(0.09)	(0.08)	(0.09)	(0.06)	(0.03)
Non-OECD	0.36	0.44	1.10	0.17	0.71	1.04
	(0.17)	(0.09)	(0.07)	(0.08)	(0.05)	(0.03)

Table 3
Investment rates are more persistent than growth rates

Notes: World = 74 countries with available data; OECD = 22 countries; and non-OECD = 52 countries. Decades consisted of the 1960s, 1970s, 1980s, and 1990s. All variables are averages over the indicated periods. Each entry is from a single regression. Bold entries indicate p-values of 1% or less. Data Sources: Barro and Lee (2000) and Penn World Table 6.1 [Heston, Summers and Aten (2002)].

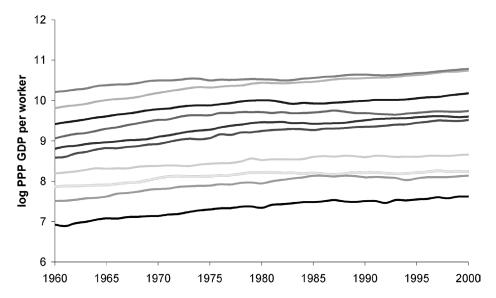


Figure 3. The evolution of income for 1960 deciles. Source: Penn World Table 6.1.

long-run growth rates, even comparing the richest and poorest countries. This sample contains 96 countries, and therefore many of the poorest countries mired in zero or negative growth.

Pritchett (1997), on the other hand, offers compelling evidence that incomes diverged massively from 1800 to 1960. Doesn't this divergence favor models, such as AK with-

	Indep	endent variable	$= s_I$	Indep	Independent variable = s_H			
	D	ependent variabl	e	Dependent variable				
	Y/L Growth rates	Y/L Log levels	# of countries	Y/L Growth rates	Y/L Log levels	# of countries		
All countries	$0.111 (0.017) R^2 = 0.32$	1.25 (0.13) $R^2 = 0.48$	96	0.210 (0.060) $R^2 = 0.15$	0.313 (0.026) $R^2 = 0.67$	74		
OECD	0.020 (0.047) $R^2 = 0.01$	0.760 (0.358) $R^2 = 0.18$	23	-0.259 x (0.078) $R^2 = 0.37$	0.119 (0.024) $R^2 = 0.56$	21		
Non-OECD	0.124 (0.023) $R^2 = 0.29$	$0.842 (0.162) R^2 = 0.28$	73	$0.367 (0.095) R^2 = 0.22$	0.314 (0.043) R2 = 0.51	53		

Table 4
Investment rates correlate more with levels than with growth rates

Notes: Variables are averages over 1960–2000. Each entry is from a single regression. Bold entries indicate *p*-values of 1% or less. Data sources: Barro and Lee (2000) and Penn World Table 6.1 [Heston, Summers and Aten (2002)].

out international externalities, in which country growth rates are not intertwined? Not necessarily. As argued by Parente and Prescott (2005), the opening up of large income differences coincided with the onset of modern economic growth. The divergence could reflect the interaction of country-specific barriers to technology adoption with the emergence of modern technology-driven growth. More generally, any given divergence episode could reflect widening barriers to importing technology rather than simply differences in conventional investment rates.

3.4. Investment rates and growth vs. levels

The AK model we sketched in the previous section predicts that a country's growth rate will be strongly related to its investment rates in physical and human capital. In Table 4 we investigate this empirically in cross-sections of countries over 1960–2000. In four of the six cases, the average investment rate is positively and significantly related to the average growth rate. For the OECD, the physical capital investment rate is not significantly related to country growth, and the human capital investment rate is actually negatively and significantly related to country growth. But for the non-OECD and all-country samples, the signs and significance are as predicted. This evidence constitutes the empirical bulwark for AK models.

In the four cases where the signs are as predicted, are the magnitudes roughly as an AK model would predict? First consider the literal AK model. According to (2.16) in the previous section, the coefficient on s_I should be A. What might be a reasonable

value for A? In order to match the average growth rate in GDP per worker (1.8%), given an average investment rate in physical capital (17%) and a customary depreciation rate (8%), the value of A would need to be

$$A = \frac{g^{\text{avg}} + \delta_K}{s_I^{\text{avg}}} = \frac{0.018 + 0.08}{0.17} \cong 0.58.$$
 (3.1)

This level of A is more than four times larger than the two significant positive coefficients on s_I in the first column of Table 4, which are around 0.12. The estimated coefficients are small in magnitude compared to what an AK model would predict. This discrepancy could reflect classical measurement error in investment rates, but such measurement error would need to account for more than 80% of the variance of investment rates across countries. Plus one would expect *positive* endogeneity bias in estimating the average level of A, due to variation in A across countries that is positively correlated with variation in s_I .

We next consider the literal BH model. According to (2.17), the coefficient on s_H should be B. To produce the average growth rate in GDP per worker given the average investment rate in human capital (8.8%) and a modest depreciation rate (2%), B would need to be

$$B = \frac{g^{\text{avg}} + \delta_H}{s_H^{\text{avg}}} = \frac{0.018 + 0.02}{.088} \cong 0.43.$$
 (3.2)

The third column of estimates in Table 4 contain coefficients on s_H . Of the two positive coefficients, one is half the predicted level (0.21) whereas the other is not far from the predicted level (0.37).

Finally, consider the hybrid model in (2.15). We assume $\gamma=0.9$ so that human capital accumulation is intensive in human capital, but does use some physical capital. For producing current output we assume the standard physical capital share of $\alpha=1/3$. We set the depreciation rates as previously mentioned. We set s_K , the share of physical capital devoted to human capital accumulation, based on (2.13). As (2.15) illustrates, we cannot independently identify A and B, only their product. We set $A^{1-\gamma}B^{1-\alpha}\cong 0.60$, so that the average predicted growth rate from (2.15) and observed s_H and s_I investment rates matches the average growth rate in GDP per worker of 1.8%. We then regress actual growth rates on predicted growth rates for a cross-section of 73 countries with available data. The coefficient estimated is 0.26 (standard error 0.08, R^2 of 0.13), far below the theoretical value of 1. Again, the empirical estimate might be low because of measurement error in predicted growth, but it would need to be large.

To recap, only 1 of the 7 coefficients of growth on investment rates considered is in the ballpark of an AK model's prediction. In contrast, we obtain uniformly positive and significant coefficients when we regress (log) *levels* of country income on country investment rates. In 5 of the 6 cases, the R^2 is notably higher with levels than with growth rates. Investment rates appear far better at explaining relative income levels than relative growth rates. The driver of growth rates would appear to be something other than simply domestic investment rates.

The preceding discussion focused on the steady-state predictions of *AK* models. It is possible that *AK* models fare better empirically when transition dynamics are taken into account. But it is worth noting that Klenow and Rodríguez-Clare (1997), Hall and Jones (1999), Bils and Klenow (2000), Easterly (2001a), Easterly and Levine (2001), and Hendricks (2002) all find that no more than half of the variation in growth rates or income levels can be attributed directly to human and physical capital. Pritchett (2005), who considers many different parameterizations of the human capital accumulation technology, likewise finds that human capital does not account for much cross-country variation in growth rates.

3.5. R&D and TFP

We now turn away from AK models to a model with diminishing returns to physical and human capital, but with R&D as another form of investment. Such a model might be able to explain country growth rates with no reference to cross-country externalities. For example, perhaps a variant of the Romer (1990) model could be applied country by country, with no international knowledge flows. R&D investment would have to behave in a way that leads to a worldwide growth slowdown, beta convergence in the OECD, and low persistence of growth rate differences. And, more directly, R&D investment would have to explain country growth rates. Research effort, like human capital, is difficult to measure. But Lederman and Saenz (2003) have compiled data on R&D spending for many countries. We now ask the same questions of their R&D investment rates that we asked of investment rates in physical and human capital: how correlated are R&D investment rates with country growth rates and country income levels?

The first column in Table 5 says that countries with high R&D spending relative to GDP do not grow systematically faster. Countries with high R&D shares do, however, tend to have high relative incomes. But the correlation with income is not significant outside the OECD. One possibility is that these regressions do not adequately control for the contributions of physical and capital. We therefore move to construct Total Factor Productivity (TFP) growth rates and levels. We subtract estimates of human and physical capital per worker from GDP per worker:

$$\ln TFP = \ln(Y/L) - \alpha \ln(K/L) - (1 - \alpha) \ln(H/L) \tag{3.3}$$

where Y is real GDP, L is employment, K is the real stock of physical capital, and H is the real stock of human capital. We suppress time and country subscripts in (3.3) for readability. We would prefer to let α vary across countries and across time based on factor shares, but such data is not readily available for most countries in the sample. We instead set $\alpha = 1/3$ for all countries and time periods. Gollin (2002) finds that capital's

⁶ Because R&D data was not available for all country-years between 1960 and 2000, we took time effects out of the variables (growth rates, income levels, investment rates in R&D), then averaged the residuals over time.

	D	ependent variabl	e	D	ependent variable	e
	Y/L Growth rates	Y/L Log levels	# of countries	TFP Growth rates	TFP Log levels	# of countries
All countries	0.40 (0.59) $R^2 = 0.01$	0.69 (0.23) $R^2 = 0.10$	82	0.43 (0.52) $R^2 = 0.01$	$0.37 (0.08) R^2 = 0.27$	67
OECD	-0.15 (0.46) $R^2 = 0.01$	$0.42 (0.11) R^2 = 0.45$	21	-0.16 (0.32) $R^2 = 0.01$	0.17 (0.06) $R^2 = 0.28$	21
Non-OECD	0.88 (1.03) $R^2 = 0.01$	0.55 (0.41) $R^2 = 0.03$	61	0.85 (1.01) $R^2 = 0.02$	0.34 (0.14) R2 = 0.12	46

Table 5

R&D intensity also correlates more with levels than growth rates

Notes: Variables are country averages over years in 1960-2000 with data relative to time effects. Y/L is GDP per worker. TFP nets out contributions from human and physical capital, as described in the text. Each entry is from a single regression. Bold entries indicate p-values of 2% or less. Data sources: Barro and Lee (2000), Penn World Table 6.1 [Heston, Summers and Aten (2002)], and Lederman and Saenz (2003).

share varies from 0.20 to 0.35 across a sample of countries, but does not correlate with country income levels or growth rates. We use Penn World Table 6.1 data assembled by Heston, Summers and Aten (2002) for PPP GDP, employment, and PPP investment in physical capital. We assume an 8% geometric depreciation rate and the usual accumulation equation to cumulate investment into physical capital stocks. We approximate initial capital stocks using the procedure in Klenow and Rodríguez-Clare (1997, p. 78). We let human capital per worker be a simple Mincerian function of schooling:

$$H = hL = \exp(\phi s)L. \tag{3.4}$$

Here h represents human capital per worker, and s denotes years of schooling attainment. We use Barro and Lee (2000) data on the schooling attainment of the 25 and older population. This data is available every five years from 1960 to 2000, with the last year an extrapolation based on enrollment rates and the slow-moving stock of workers. A more complete Mincerian formulation would include years of experience in addition to schooling and would sum the human capital stocks of workers with different education and experience levels. In Klenow and Rodríguez-Clare (1997) we found that taking experience and heterogeneity into account had little effect on aggregate levels and growth rates, so we do not pursue it here. We use (3.4) with the Mincerian return $\phi = 0.085$, based on the returns estimated for many countries and described by Psacharopoulos and Patrinos (2002).

The latter columns in Table 5 present regressions of TFP growth rates and levels on R&D investment rates. The sample of countries is smaller given data limitations (67 countries rather than 82). Just like growth in GDP per worker, growth in TFP is not significantly related to R&D investment rates. But TFP levels, like levels of GDP per worker, are positively and significantly related to R&D investment rates. From this we conclude that even R&D investment rates affect relative income levels, not long-run growth rates. The persistence of R&D investment rate differences across countries, combined with the lack of persistent growth rate differences, supports this interpretation. We are led to consider models in which country growth rates are tethered together.

Before considering a model with international knowledge externalities, we pause to consider a model with "externalities" operating through the terms of trade. We have in mind Acemoglu and Ventura's (2002) model of the world income distribution. In their model, each country operates an AK technology, but uses it to produce distinct national varieties. Countries with high AK levels due to high investment rates plentifully supply their varieties, driving down their prices on the world market. This results in a pAK model with a stationary distribution of income even in the face of permanent differences in country investment rates (and A levels, for that matter). Prices tether incomes together in the world distribution, not the flow of ideas. This is a clever and coherent model, but we question its empirical relevance. Hummels and Klenow (2005) find that richer countries tend to export a given product at higher rather than lower prices. They do estimate modestly lower quality-adjusted prices for richer countries, but nowhere near the extent needed to offset AK forces and generate "only" a factor of 30 difference in incomes.

To summarize this section, AK models tightly connect investment rates and growth rates. Such a tight connection does not hold empirically. This is the case for the world growth slowdown, for OECD convergence, for growth persistence, and for country variation in growth vs. income levels. A version of the AK model with endogenous terms of trade might be able to circumvent these empirical hazards but faces empirical troubles of its own. We therefore turn to models with international knowledge externalities that drive long-run growth.

4. Models with common growth driven by international knowledge spillovers

Based on evidence in the previous section, we now focus on models with two features. The first is that, in steady state, all countries grow at the same rate thanks to international knowledge spillovers. The second feature is that differences in policies or other country parameters generate differences in TFP levels rather than growth rates. Examples of this type of model are Howitt (2000), Parente and Prescott (1994), and Eaton and Kortum (1996, 1999), as well as the model of technology diffusion in Chapter 8 of Barro and Sala-i-Martin (1995). Eaton and Kortum (1999) is a particularly close predecessor to our approach, although they go farther in tying their analysis to international patenting data in the G5 countries.

In these models there is a world technology frontier, and a country's research efforts determine how close the country gets to that frontier. There are three different issues that must be addressed. First, what determines the growth rate of the world technology frontier? Second, how is it that a country's research efforts allow it to "tap into" the world technology frontier? And third, what explains differences across countries in their research efforts? Our goal in this section is to build on the ideas developed in the recent literature to construct a model that offers a unified treatment of these three issues and that is amenable to calibration. The calibration is intended to gauge the model's implications about the strength of the different externalities and the drivers of cross-country productivity differences.⁷

To highlight the different issues relevant for the model, our strategy is to present it in parts. The next subsection 4.1 takes world growth and R&D investment as exogenous and discusses how R&D investment determines steady state relative productivity. Subsection 4.2 discusses different ways of modeling how world-wide R&D investment determines the growth rate of the world technology frontier. Subsection 4.3 extends the model so as to allow for endogenous determination of countries' R&D investment rates. Subsection 4.4 calibrates the model. Finally, subsection 4.5 presents the results of an exercise where we calculate, for each country in our sample, the impact on productivity from international spillovers.

4.1. R&D investment and relative productivity

In this section we focus on a single country whose research efforts determine its productivity relative to the world technology frontier. Both the R&D investment rate and the rate of growth of the world technology frontier are exogenous. Output is produced with a Cobb–Douglas production function: $Y = K^{\alpha}(AhL)^{1-\alpha}$, where Y is total output, K is the physical capital stock, A is a technology index, h is human capital per person, and L is the total labor force. We assume that h is constant and exogenous. Output can be used for consumption (C), investment (I), or research (R), Y = C + pI + R, where p is the relative price of investment and is assumed constant through time. Capital is accumulated according to: $\dot{K} = I - \delta K$. Finally, A evolves according to:

$$\dot{A} = (\lambda R/L + \varepsilon A)(1 - A/A^*) \tag{4.1}$$

where λ is a positive parameter and A^* is the world technology frontier, both common across countries.⁸

There are three salient differences between this model and the standard endogenous growth model. Firstly, the productivity of research in generating A-growth is affected by

⁷ Although we refer to research externalities throughout this section, the knowledge externalities could just as well be with respect to human capital. Only when we use data on R&D is the analysis specific to research.

⁸ In models like those of Parente and Prescott (1994) and Howitt (2000) research is meant to capture both R&D and technology adoption efforts. In this paper we follow this practice and simply refer to the sum of these two technology investments as R&D or just "research".

the country's productivity relative to the frontier, as determined by the term $(1 - A/A^*)$ in (4.1). This captures the idea that there are "benefits to backwardness". One reason for this may be that the *effective cost* of innovation and technology adoption falls when a country is further away from the world technology frontier. This is what happens in Parente and Prescott (1994) and in Barro and Sala-i-Martin (1995, Chapter 8). Alternatively, being further behind the frontier may confer an advantage because every successful technology adoption entails a *greater improvement* in the national technology level. This is what happens in Howitt (2000) and in Eaton and Kortum (1996).

Secondly, we introduce $\varepsilon \geqslant 0$ to capture the sources of technology diffusion from abroad that do not depend on domestic research efforts. We have in mind imports of goods that embody technology, and that do not require upfront adoption costs (e.g., equipment which is no harder to use but which operates more efficiently). ¹⁰ As we will see below, this is important for the model to match certain features of the data.

Thirdly, in contrast to most endogenous growth models, we divide research effort by L in the A-growth expression above. This is done to get rid of scale effects and can be motivated in two ways. First, if A represents the quality of inputs, then one can envisage a process where an increase in the labor force leads to an expansion in the variety of inputs [Young (1998) and Howitt (1999)]. With a larger variety of inputs, research effort per variety is diluted. This eliminates the impact of L on A growth. Second, if research is undertaken by firms to increase their own productivity, then population growth may lead to an expansion in the number of firms and a decrease in the impact of aggregate research on firms' A-growth [Parente and Prescott (1994)]. In this case, L represents the number of firms.

The measured R&D investment rate is given by $s_R = R/Y$. This implies that $R/(AL) = s_R Y/(AL) = s_R k$ where $k \equiv (K/Y)^{\alpha/(1-\alpha)}h = Y/(AL)$. To proceed, note that in steady state $a \equiv A/A^*$ will be constant, since A will grow at the same rate as A^* , which we denote by g_A . Thus, from (4.1)

$$g_A = (\lambda s_R k + \varepsilon)(1 - a). \tag{4.2}$$

Solving for a we obtain:

$$a = 1 - \frac{g_A}{\lambda s_R k + \varepsilon}. (4.3)$$

The values of k and s_R determine a country's relative A from (4.3). Conceivably, the parameter λ (TFP in research, if you will) could differ across countries and also contribute to differences in A. But in this paper we assume λ does not vary across countries. We

⁹ In Howitt's model, $(1 - A/A^*)$ arises from the product of two terms: $(1/A^*)(A^* - A)$. The $(1/A^*)$ term arises because, as the world's technology becomes more advanced, more research is required to tap into it; the second term captures the fact that, when the country is more backward, every successful technology adoption entails a *greater improvement* in the national technology level.

¹⁰ This free flow of ideas is also likely to depend on the local presence of multinationals, which bring valuable knowledge that diffuses to other local firms without the need for additional R&D.

do, however, allow researchers to be more productive in countries with more physical and human capital per worker.

The previous results clearly show that policies that lower investment in physical or human capital or R&D do not affect a country's growth rate. Their effect is on a country's steady state relative A. Also, as discussed above, there are no scale effects in this model: higher L does not lead to higher growth or to a higher relative A. This stands in contrast to most growth models based on research [e.g., Romer (1990), Barro and Sala-i-Martin (1995, Chapter 8)].

It is also noteworthy in Equation (4.3) that the value of k, which captures physical and human capital intensity, affects a country's TFP level conditional on its R&D investment rate. Thus, large differences in TFP across countries do not necessarily imply that differences in human and physical capital stocks are just a small part of cross country income differences. Indeed, this model suggests that some of the TFP differences may be due to differences in capital intensities across countries. Below we explore this issue quantitatively.

It is instructive to calculate the social rate of return to research at the national level. As shown in Jones and Williams (1998), this can be done even without knowing the details of the model that affect the endogenous determination of the R&D investment rate. Letting $\dot{A} = G(A, R)$, Jones and Williams show that the (within-country) social rate of return \tilde{r} can be expressed as:

$$\tilde{r} = \frac{\partial Y/\partial A}{P_A} + \partial G/\partial A + g_{P_A}.$$
(4.4)

Here P_A stands for the price of ideas and is given by $P_A = (\partial G/\partial R)^{-1}$. As explained by Jones and Williams, the first two terms in (4.4) represent the dividends of research while the third term represents the associated capital gains. The first dividend term is the obvious component, namely the productivity gain from an additional idea divided by the price of ideas. The second dividend term captures how an additional idea affects the productivity of future R&D.

In the model we presented above, it is straightforward to show that, along a steady state path, we have:

$$\tilde{r} = (1 - \alpha)\lambda k(1 - a) + \left[\varepsilon(1 - a) - \frac{ag_A}{1 - a}\right] + g_L. \tag{4.5}$$

The first term on the right-hand side corresponds to the first dividend term in Jones and Williams' formula. The second term, in square brackets, corresponds to the indirect effect of increasing A on the cost of research $(\partial G/\partial A)$. The third term, g_L , corresponds to the term capturing the capital gains in Jones and Williams formula. To understand this last term, note that we have implicitly assumed that new varieties or firms start up with the same productivity as existing varieties or firms. Thus, the value of ideas will rise faster with a higher g_L , and the social return to research will correspondingly increase with g_L .

Also note that, since the RHS of (4.5) is decreasing in a, and a is increasing in s_R , the social rate of return to research will be decreasing in s_R , as one would expect. If k varies less than in the data, one should also expect to find higher social rates of return to research in poor countries than in rich countries, as found by Lederman and Maloney (2003).

More importantly, if ε is close to zero, then from (4.2) and (4.5) we should have $\tilde{r} \approx (1 - \alpha)\lambda k(1 - a) \approx (1 - \alpha)g_A/s_R$. Using the growth rate of A in the OECD in the period 1960–2000 as an approximation of g_A (1.5%), and using $\alpha = 1/3$, then $\tilde{r} \approx 0.01/s_R$. Noting that the median of s_R in the non-OECD countries we have in our sample is $s_R = 0.5\%$, then $\tilde{r} \geq 200\%$. This seems implausibly high. There are two ways out of this problem. First, one can argue that measured R&D investment does not capture all the research efforts undertaken by countries. Clearly, higher R&D investment rates would lead to lower and more plausible social rates of return to research. Second, one can argue that the implausible implications of the model are due to the assumption that ε is close to zero. In the calibration exercise in Section 4.4, we will argue that both of these solutions are needed to make the model consistent with the data.

4.2. Modeling growth in the world technology frontier

In this section we extend the model so that g_A is endogenously determined by the research efforts in all countries. The models we mentioned above deal with this in different ways, except Parente and Prescott who leave g_A as exogenous. Barro and Sala-i-Martin (1995, Chapter 8) have a Romer-type model of innovation that determines g_A in the "North". We do not pursue this possibility because of the scale effect that arises in their model (larger L in the North leads to higher g_A) and because we want to allow research efforts by all countries to contribute to the world growth rate. We first consider an adaptation of Howitt's (2000) formulation. A country's total effective research effort, λR_i , gets diluted by the country's number of varieties or number of firms, both represented by L_i , and is then multiplied by a common spillover parameter, σ , to determine that country's contribution to the growth of the world's technology frontier:

$$\dot{A}^* = \sigma \sum_{i} \left(\frac{\lambda R_i}{L_i} \right).$$

Given our results above, we obtain:

$$g_A = \sigma \sum_i \lambda k_i s_{Ri} a_i. \tag{H1}$$

This formulation has the nice feature that the world growth rate does not depend on the world's level of L (no scale effect on growth at the world level), although it does depend

¹¹ The problem is not so pronounced for the U.S. Given its measured R&D investment rate of $s_R = 2.5\%$, we have $\tilde{r} \approx 40\%$, which is in the range of estimates of the social rate of return to R&D in the U.S. See Griliches (1992) and Hall (1996).

positively on R&D investment rates. The main problem with this formulation, and the reason we do not pursue it further, is that larger countries contribute no more to world growth than smaller countries do. This has the implausible implication that subdividing countries would raise the world growth rate.

In footnote 22 of his paper, Howitt discusses an alternative specification wherein country spillovers are diluted by world variety rather than each country's variety. This implies that:

$$\dot{A}^* = \sigma \sum_{i} \left(\frac{\lambda R_i}{L} \right).$$

where $L = \sum L_i$. Howitt does not pursue this approach because, in the presence of steady-state differences in the rate of growth of L across countries, g_A would be completely determined in the limit by the research effort of the country with the largest rate of growth of L. We believe, however, that it is quite natural to analyze the case in which g_L is the same across countries. ¹² In this case, $\omega_i \equiv L_i/L$ is constant through time, and the expression above can be manipulated to yield:

$$g_A = \sigma \sum_i \lambda k_i s_{Ri} a_i \omega_i. \tag{H2}$$

If we think of L as the number of firms rather than the number of varieties of capital goods, then (H2) amounts to stating that g_A is determined by the country-workforce-weighted average research intensity across firms world-wide. This seems much more reasonable than (H1), where g_A is determined by the *unweighted* average of research intensity across countries.

Expression (H2) differs from (H1) only in the presence of the weights ω_i that represent shares of world L. This has two advantages: first, large countries contribute more to world growth than small countries do, and second, subdividing countries would not affect the world growth rate. But (H2) has a problematic implication, namely that those countries with higher than average $k_i s_{Ri} a_i$ would be better off disengaging from the rest of the world – their growth rate would be higher if they were isolated. That is, a research intensive country would be better off ignoring the research done in other countries.

According to Howitt's variety interpretation of this model, this is because an isolated country's growth rate would be given by $\sigma \lambda k_i s_{Ri} a_i$. Its research intensity would no longer be spread out over the number of world varieties, but instead over the smaller number of the country's own varieties. Thus, when a country disengages, it no longer benefits from spillovers from research conducted by the rest of the world, but there is

¹² If one country's population did come to dominate world population, however, it might be sensible to say it does almost all of the world's research and, hence, it will virtually determine the world growth rate. We assume equal labor force growth rates across countries not because we think it is accurate for describing what is happening now, but because we think it is a convenient fiction for a steady state model to explore international spillovers.

an important compensating gain that comes from the fact that variety – and therefore dilution – falls for the disengaging country. Since there is no love of variety in Howitt's model, a high research-intensity country would gain from disengagement. By this logic, engagement could not be sustained among any set of asymmetric countries! The higher $k_i s_{Ri} a_i$ countries would always prefer to disengage, leaving all countries isolated in equilibrium.

We now turn to an alternative specification for world spillovers in which variety does not play such a crucial role. The specification will exhibit several of the features we have been looking for: first, no scale effect of world population on the world's growth rate; second, other things equal, larger countries contribute more to world prosperity than small countries do; and third, tapping into rest-of-world research does not require spreading research across more varieties. We believe this is accomplished by adopting the formulation in Jones (1995): instead of dividing by L, the scale effect is avoided by introducing the assumption that advancing the world technology frontier gets harder as the frontier gets higher. This can be captured by the following specification of international spillovers:

$$\dot{A}^* = \left(A^*\right)^{\gamma - 1} \sigma \sum_i \lambda R_i \tag{4.6}$$

where $\gamma < 1$. In this setting, sustained growth in A^* depends on a continuously rising population. To see this, notice that we can restate (4.6) as follows:

$$g_A = (A^*)^{\gamma - 1} L\sigma \sum_i \lambda k_i s_{Ri} a_i \omega_i.$$
 (J)

This expression makes clear that g_A is decreasing in A^* ; as mentioned above, this is what is going to eliminate the scale effect. Since all of the terms in the summation on the right-hand side of (J) are constant, then – differentiating with respect to time – we get that:

$$g_A = \frac{g_L}{1 - \gamma}.\tag{4.7}$$

One criticism of this specification is that g_A does not depend on s_R , hence policy-induced increases in research intensity would not increase the world's growth rate [Howitt (1999)]. As Jones (2002) argues, however, research intensity has been increasing over the last decades without a concomitant increase in the growth rate, so it is far from clear that we want a model where g_A depends on s_R .¹³

An interesting and relevant feature of the model presented by Eaton and Kortum (1996) is that it allows for spillovers to differ between pairs of countries. We can introduce this feature in the model by doing two things: first, we allow each country to

¹³ Even though research intensity does not affect the growth rate, it can have sizable effects on welfare, particularly when – as evidence suggests – the social rate of return of research is significantly higher than the private rate of return.

have a different technology frontier, A_i^* ; second, we add country-pair specific spillover parameters, η_{il} , to (4.6) so that now:

$$\dot{A}_i^* = \left(A_i^*\right)^{\gamma - 1} \sigma \sum_{l} \lambda R_l \eta_{il}.$$

This formulation implies that there will no longer be a world technology frontier in the way it existed in model (J). However, it proves useful for the analysis to introduce a new concept, which we will denote by \tilde{A} and which could be understood as the "frictionless technology frontier". To define this concept, note that if spillovers were the same among all country pairs $(\eta_{il}=1 \text{ for all } i \text{ and } l)$ – a case we could interpret as frictionless – then countries would have a *common* technology frontier: $A_i^*=A_l^*$ for all i and l. We define \tilde{A} so that in this case $(\eta_{il}=1 \text{ for all } i \text{ and } l)$ $A_i^*=\tilde{A}$ for all i. As we will see below, in steady state \tilde{A} grows at the same rate as A_i^* for all i. Letting $z_i\equiv A_i^*/\tilde{A}$, which captures the strength of spillovers from the rest of the world to country i, we arrive at the following steady state restriction:

$$g_A = \left(A_i^*\right)^{\gamma - 1} L(\sigma/z_i) \sum_{l} \lambda k_l s_{Rl} a_l \omega_l z_l \eta_{il}$$
 (JEK)

where JEK stands for Jones, Eaton and Kortum and where a_l is now country l's technology level relative to *its own* technology frontier: $a_l \equiv A_l/A_l^*$. It can be shown that this implies the following restriction for \tilde{A} :

$$\tilde{A} = (vL)^{1/(1-\gamma)} \tag{4.8}$$

where $v \equiv (\sigma/g_A) \sum_l \lambda k_l s_{Rl} a_l \omega_l$. It is clear that each country's technology frontier and \tilde{A} will grow at the same rate as A^* did in model (J), given by $g_L/(1-\gamma)$.

It is interesting to pause here to discuss the model's implications regarding the effect of country size on productivity. Imagine, to simplify, that all countries are the same except for size, and assume that $\eta_{ij}=1$ for i=j and $\eta_{ij}=\eta<1$ for $i\neq j$. Then it is easy to show that $z_i>z_j$ if $\omega_i>\omega_j$; larger countries are more productive. Intuitively, larger countries benefit more from spillovers because more of the world's research takes place within their borders. As long as borders discretely reduce spillovers, larger countries will capture more spillovers and enjoy higher productivity.

The next step is to impose some restrictions on the international spillover parameters. The literature has allowed international spillovers to depend on trade [Coe and Helpman (1995)], distance [Eaton and Kortum (1996)], and other variables such as FDI flows [Caves (1996)]. Here we focus on the simplest approach, which is to assume that the parameters η_{il} are completely determined by distance. (This would capture trade and FDI related spillovers that are related to distance.) We do this by assuming that $\eta_{il} = e^{-\theta d(i,l)}$, where d(i,l) is bilateral distance between countries i and l, and θ is some positive parameter. This model collapses to (J) if $\theta = 0$.

This completes our discussion of different ways to model international spillovers. Table 6 summarizes the discussion in this subsection.

Spillovers Growth rate Advantages Disadvantages $\dot{A}^* = \sigma \sum_{i} \left(\frac{\lambda R_i}{L_i} \right)$ $g_A = \\ \sigma \sum_i \lambda k_i s_{Ri} a_i$ No scale effects Larger countries contribute no more to g_A than do small countries H2 $\dot{A}^* = \sigma \sum_i \left(\frac{\lambda R_i}{L}\right)$ Previous ones plus: Countries with higher than $g_A =$ Previous ones plus $\sigma \sum_i \lambda k_i s_{Ri} a_i \omega_i$ Size matters for a average $k_i s_{Ri} a_i$ would be country's contribution better off ignoring $\omega_i = L_i/L$ research from the rest of to g_A the world J $\dot{A}^* = (A^*)^{\gamma - 1} \sigma \sum_i \lambda R_i$ $g_A = g_L/(1 - \gamma)$ Previous ones plus: gA does not depend on Research-intensive R&D efforts ... but is this countries do not a disadvantage? [See prefer to disengage Jones (1995).] from the rest of the world. JEK $\dot{A}_i^* = (A_i^*)^{\gamma - 1} \sigma \sum_l \lambda R_l \eta_{il}$ $g_A = g_L/(1 - \gamma)$ Previous ones plus: We will find it hard to see The model takes into the cost of geographic account effect of isolation in the TFP data.

Table 6
Alternative ways of modeling international spillovers

4.3. Determinants of R&D investment

We mentioned above that there are two ways to motivate the model we presented in subsection 4.1. First, we can think of a model like the one presented in Howitt (2000), where research leads to improvements in the quality of capital goods, and population growth leads to an expansion in the total number of varieties available. Second, research may be carried out by firms to increase their own productivity, as in Parente and Prescott (1994). We pursue this second approach because it is simpler and much more convenient for our calibration purposes later on.

distance on spillovers.

As in Parente and Prescott (1994), we assume a constraint on the amount of labor firms can hire. In particular, we assume that firms can hire no more than F workers. To simplify notation, we set F=1. This constraint can be motivated as a limitation on the span of control by managers, as in Lucas (1978). Output produced by firm j in country i at time t, which we denote by Y_{jit} , is given by $Y_{jit} = K^{\alpha}_{jit}(A_{jit}h_i)^{1-\alpha}$. (We now use time subscripts because they clarify the maximization problem below.) The firm can convert output into consumption, investment goods or R&D according to $Y_{jit} = C_{jit} + p_i I_{jit} + R_{jit}$, and the firm's capital stock evolves according to $K_{jit} = C_{jit} + C_{j$

¹⁴ If one takes F = 1 literally, then the externalities are in the human capital investment of individual workers.

 $I_{jit} - \delta K_{jit}$. Finally, the firm's technology index A_{jit} evolves according to:

$$\dot{A}_{jit} = \left((1 - \mu)\lambda R_{jit} + \mu \lambda \overline{R}_{it} + \varepsilon A_{jit} \right) \left(1 - A_{jit} / A_{it}^* \right) \tag{4.9}$$

where μ is a parameter between zero and one, \overline{R}_{it} is the average of R_{jit} across firms in country i (we use the bar over the variable to emphasize that this is the average across firms, and not the aggregate economy-wide variable), and A_{it}^* is the technology frontier for country i with $\dot{A}_{it}^*/A_{it}^* = g_A$ for all i in steady state.

There are two features in this specification that merit some explanation. First, the "benefits of backwardness" are determined by the term $1 - A_{jit}/A_{it}^*$, which can differ across firms in country i: a more backward firm in country i would have a higher catchup term. If instead we specified the catch-up term as $1 - \overline{A}_{it}/A_{it}^*$ (where A_{it} is the average technology index across firms in country i), then there would be a negative externality because, as a firm does more research, it increases the country's average technology index and decreases the catch-up term for the other firms. Given that there is no particular reason to think that this negative externality is a relevant feature to include in the model, we have chosen to specify the catch-up term as $1 - A_{jit}/A_{it}^*$. Second, there is a positive research externality across firms within each country, represented by the term $\mu\lambda\overline{R}_{it}$. This externality captures the idea that a firm benefits directly from research undertaken by other firms within the same economy.

To relate this to what we had in subsection 4.1, note that if firms within a country are identical, then $R_{jit} = \overline{R}_{it}$ and $A_{jit} = \overline{A}_{it}$. Using this in (4.9), we obtain:

$$\dot{\overline{A}}_{it} = (\lambda \overline{R}_{it} + \varepsilon \overline{A}_{it})(1 - \overline{A}_{it}/A_{it}^*).$$

But note that $A_{it} = \overline{A}_{it}$ and $\overline{R}_{it} = R_{it}/L_{it}$, where L_{it} is the total labor force in country i and also the number of firms there, given our assumptions above. Using these results and noting that $S_{Ri} = R_{it}/Y_{it}$ we obtain Equation (4.2).

Firms in country i pay taxes at rate τ_{Ki} on capital income (output less the wage bill), and there is an R&D tax (or subsidy, if it is negative) of τ_{Ri} .¹⁵ We stress that this R&D tax parameter does not have to be interpreted strictly as a formal tax or subsidy; when positive, the R&D tax parameter τ_{Ri} could also be interpreted as capturing "barriers to technology adoption", as in Parente and Prescott (1994).¹⁶

The firm's dynamic optimization problem is to choose a path for R_{jis} and to maximize

$$\int_{t}^{\infty} \left((1 - \tau_{Ki}) [Y_{jis} - w_{is}] - p_{i} I_{jis} - (1 + \tau_{Ri}) R_{jis} \right) e^{-r(s-t)} ds$$

¹⁵ We should note here that the tax rate on capital income also affects the incentive to do research. The notation used for the two tax rates is meant to emphasize that τ_{Ki} affects all forms of accumulation by the firm, whereas τ_{Ri} only affects research expenditures.

¹⁶ We assume that any tax revenue collected is distributed back to consumers in lump-sum fashion.

subject to $\dot{K}_{jis} = I_{jis} - \delta K_{jis}$, $\dot{A}_{is}/A_{is} = \dot{A}_{is}^*/A_{is}^* = g_A$, and

$$\dot{A}_{jis} = ((1-\mu)\lambda R_{jis} + \mu\lambda \overline{R}_{is} + \varepsilon A_{jis})(1 - A_{jis}/A_{is}^*).$$

As shown in Appendix A, by imposing the symmetry condition on the two Euler equations for this optimization problem, we obtain the following two conditions for the symmetric equilibrium:

$$\frac{p_i K_{it}}{Y_{it}} = \alpha \frac{1 - \tau_{Ki}}{r + \delta},\tag{4.10}$$

$$\Omega_i(1-\alpha)\lambda k_i(1-a_i) - g_A a_i/(1-a_i) + \varepsilon(1-a_i) = r$$
 (4.11)

where

$$\Omega_i \equiv \frac{(1 - \tau_{Ki})(1 - \mu)}{(1 + \tau_{Ri})}.$$

Equation (4.10) defines the equilibrium capital-output ratio in country i and Equation (4.11) implicitly defines the equilibrium relative A in country i. Given a_i and knowing k_i from the data, we can plug their values into Equation (4.3) to obtain the equilibrium steady state R&D investment rate, s_{Ri} . It is easy to see that an increase in the capital income tax or the R&D tax or an increase in the externality parameter, μ , would decrease Ω_i and hence lead to a decline in equilibrium a_i (this is because the left-hand side of (4.11) is decreasing in a_i). This, of course, would imply a decline in the R&D investment rate. The same reasoning shows that a_i is increasing in k_i but it is not necessarily the case that s_{Ri} increases with k_i (see Appendix A).

Combining the result for the social rate of return in Equation (4.5) with (4.11), we obtain the following expression for the wedge between the social and private rate of return to R&D:

$$\tilde{r}_i - r = (1 - \Omega_i)(1 - \alpha)\lambda k_i(1 - a_i) + g_L. \tag{4.12}$$

The first term on the right-hand side is the distortion created by Ω , which captures the effect of the income tax, τ_K , the R&D tax, τ_R , and the externality parameter, μ . If there are no taxes and $\mu=0$ (no domestic R&D externalities), then $\Omega_i=1$ and the wedge between the social and private rate of return to R&D collapses to g_L .¹⁷

4.4. Calibration

The model described in the previous section, together with the (JEK) formulation for international spillovers with $\eta_{il} = \mathrm{e}^{-\theta d(i,l)}$, constitutes the model we calibrate in this subsection. Since we will only be working with the symmetric steady state equilibrium,

 $^{^{17}}$ As explained above, g_L is associated with a positive externality because new firms start up with the same productivity as existing firms. Since the number of firms is equal to the workforce, the value of ideas and the social rate of return are increasing in g_L .

in this subsection we suppress time and firm subscripts to simplify notation. Given N countries, the steady state equilibrium is given by $\{K_i, Y_i, k_i, a_i, s_{Ri}, A_i, A_i^*, z_i; i = 1, ..., N\}$ such that:

$$g_A = \frac{g_L}{1 - \nu},\tag{4.13}$$

$$\frac{p_i K_i}{Y_i} = \alpha \frac{1 - \tau_{Ki}}{r + \delta},\tag{4.14}$$

$$k_i = h_i \left(\frac{K_i}{Y_i}\right)^{\alpha/(1-\alpha)},\tag{4.15}$$

$$\Omega_i(1-\alpha)\lambda k_i(1-a_i) - \frac{g_A a_i}{1-a_i} + \varepsilon(1-a_i) = r,$$
(4.16)

$$a_i = 1 - \frac{g_A}{\lambda s_{Ri} k_i + \varepsilon},\tag{4.17}$$

$$A_i = a_i A_i^*, (4.18)$$

$$A_i^* = z_i \tilde{A},\tag{4.19}$$

$$\tilde{A} = (vL)^{1/(1-\gamma)},$$
(4.20)

$$v = \frac{\sigma}{g_A} \sum_{l} \lambda k_l s_{Rl} a_l \omega_l, \tag{4.21}$$

$$z_i^{(1-\gamma)+1} = \frac{\sigma}{vg_A} \sum_{l} \lambda k_l s_{Rl} a_l \omega_l z_l e^{-\theta d(i,l)}$$
(4.22)

where the last equation comes from (JEK) together with (4.8).

If we knew the relevant parameters and tax rates and wanted to solve for an equilibrium, we would first start by solving for g_A from Equation (4.13). Given data for exogenous variables h_i , p_i and τ_{Ki} we could then calculate equilibrium k_i using (4.14) and (4.15). Together with g_A and parameter ε , Equation (4.16) would yield a_i . From (4.17) we would then obtain s_{Ri} . Up to this point, there is no interaction across countries, so these results do not depend on geography or θ ; this dimension becomes relevant in obtaining actual productivity levels, because they depend on the variables z_i , which capture spillovers from the rest of the world to country i. To see how this operates, note that given the value of θ , Equation (4.22) configures a system of N equations (where N is the number of countries) in N unknowns (z_1, z_2, \ldots, z_N) . The solution to this system determines z_i . Given parameter σ , Equation (4.20) determines \tilde{A} , which together with z_i determines each country's technology frontier A_i^* (Equation (4.19)). Finally, from Equation (4.18), a country's technology frontier together with its relative A level a_i determines A_i .

For the calibration exercise, the first step is to specify the variables we observe and how they relate to the model. We take human capital to be $h_i = e^{\varphi^* MYS_i}$, where MYS_i is mean years of schooling of the adult population in country i, obtained from Barro and Lee (2000). We use R&D data from Lederman and Saenz (2003). The 48 countries in our sample are the ones for which there is R&D data for 1995, as well as the necessary

TFP and capital intensity variables described in Section 3. The first two columns of Table 7 reproduce the values of the R&D investment rate and the value of A for the 48 countries in our sample.

For the basic parameters we use the following values: $\varphi = 0.085$, $\alpha = 1/3$, $\delta = 0.08$, $g_L = 0.011$ and $g_A = 0.015$. For the first three, see our discussion in Section 3. The last two (the growth rates) were obtained from OECD average growth rates of L and A for the period 1960–2000. Wising (4.13), the values for the two growth rates imply $\gamma = 0.31$. To calculate the net private rate of return, r, which we assume to be common across countries, we take the capital income tax in the U.S. to be 25% ($\tau_{K, \text{US}} = 0.25$). Given the 1995 U.S. nominal capital–output ratio of 1.5 (see Section 3 for how we constructed capital–output ratios), this implies from (4.14) that r = 8.6%. Given this level for r, we then use Equation (4.14) together with country nominal capital–output ratios to obtain each country's implicit income tax τ_{Ki} .

Remaining parameters we must calibrate are ε , λ , μ and θ . Unfortunately, there is no empirical work that we can rely on to pin down ε . Thus, we choose a value for ε based on the following reasoning. First, ε cannot be much higher than g_A . This is because for $ks_R \ge 0$ Equation (4.17) implies that $a \ge 1 - g_A/\varepsilon$. Thus, a high value of ε would imply that some countries' relative empirical A becomes lower than the theoretical minimum $1 - g_A/\varepsilon$. In other words, if free technology diffusion is too important, then it would be hard to account for countries with very low A levels. Second, if $\varepsilon < g_A$, then countries with a low value of ks_R ($\lambda s_R k < g_A - \varepsilon$) would not be able to keep up with the world's rate of growth in technology, so they would not have a steady state relative A level. (Consistent with stable long run relative income levels, Figure 3 showed roughly parallel slopes for average income across deciles over 1960–2000, with each decile based on 1960 income.) Thus, it seems reasonable to impose the intermediate condition that $\varepsilon = g_A$. We believe, however, that future empirical work should attempt to understand the importance of free technology diffusion captured by parameter ε .

Given this choice for ε , we use two empirical findings to pin down parameters λ and μ , namely that the social rate of return to R&D in the U.S. is three times the net private rate of return [Griliches (1992)] and that the U.S. imposes a subsidy of 20% on R&D [Hall and Van Reenen (2000)], implying that $\tau_{R, \text{US}} = -0.2.^{20}$ Given data for s_R and k for the U.S. in 1995 ($s_{R, \text{US}} = 2.5\%$ and $k_{\text{US}} = 3.6$), then this restriction together with Equation (4.17) implies $a_{\text{US}} = 0.7$ and $\lambda = 0.38.^{21}$ From (4.16) we then obtain $\mu = 0.55$.

¹⁸ Specifically, the growth rate of *A* is the annual growth rate of the weighted average of *A* in the OECD with weights given by employment levels in 1960. OECD membership is defined by 1975 status.

¹⁹ Auerbach (1996) estimates an effective tax rate in the U.S. of about 16%, but King and Fullerton (1984) estimate a much higher level of around 35%. We use 25% as an intermediate value.

²⁰ The 20% subsidy on R&D is the statutory rate, but not necessarily the effective one. Bronwyn Hall suggested that a 5% subsidy would be a better approximation to the effective rate. We performed an alternative calibration with $t_{R,US} = -0.05$ and the results did not change significantly.

²¹ Due to the non-linearity of the expression for the social rate of return to R&D, there are actually two values of λ which are compatible with a social rate of return equal to 26% (three times the private rate of return).

 $\label{thm:continuity} Table \ 7$ Data and "true" values for research intensity and productivity

Country	Data s_R	Data A	"True" s _R	Implied A
Argentina	0.41%	9,720	1.21%	9,719
Bolivia	0.37%	4,672	0.74%	4,672
Brazil	0.86%	9,836	1.67%	9,835
Chile	0.61%	11,078	1.98%	11,075
China	0.60%	2,570	0.28%	2,570
Colombia	0.28%	8,143	1.54%	8,141
Ecuador	0.08%	5,990	0.69%	5,990
Egypt	2.11%	11,126	3.57%	11,119
Hong Kong	0.25%	17,874	5.49%	17,732
Hungary	0.73%	7,172	0.63%	7,172
Indonesia	0.09%	5,912	0.91%	5,911
India	0.63%	3,755	0.60%	3,755
Israel	2.75%	13,919	2.15%	13,922
South Korea	2.49%	8,842	0.71%	8,843
Mexico	0.31%	8,781	1.08%	8,780
Panama	0.38%	6,106	0.60%	6,106
Peru	0.05%	4,285	0.40%	4,285
Poland	0.69%	4,893	0.33%	4,893
Romania	0.80%	2,757	0.16%	2,757
Senegal	0.02%	3,069	0.64%	3,068
Singapore	1.16%	13,592	2.16%	13,587
El Salvador	0.33%	11,096	3.26%	11,084
Thailand	0.12%	5,212	0.49%	5,212
Tunisia	0.32%	10,323	2.11%	10,319
Taiwan	1.78%	14,944	3.59%	14,928
Uganda	0.59%	2,878	1.02%	2,878
Uruguay	0.28%	10,088	1.69%	10,085
Venezuela	0.48%	9,427	1.35%	9,426
Austria	1.56%	14,807	2.60%	14,800
Belgium	1.57%	15,597	2.89%	15,586
Canada	1.64%	11,614	1.12%	11,615
Denmark	1.84%		1.12%	
	0.81%	13,678 15,758	3.69%	13,677 15,726
Spain				
Finland	2.37%	10,358	0.94%	10,360
France	2.31%	15,411	3.07%	15,404
United Kingdom	1.99%	13,954	2.35%	13,952
Germany	2.25%	11,993	1.31%	11,994
Greece	0.49%	10,046	1.07%	10,046
Ireland	1.35%	17,177	5.08%	17,098
Italy	1.08%	19,204	8.27%	18,795
Japan	2.89%	9,864	0.85%	9,865
Netherlands	1.99%	14,136	2.19%	14,135
Norway	1.71%	10,990	0.88%	10,991
New Zealand	0.97%	9,911	0.85%	9,911
Portugal	0.57%	13,230	2.65%	13,220
Sweden	3.46%	10,416	0.91%	10,418
Turkey	0.38%	7,800	1.18%	7,800
USA	2.51%	15,472	2.51%	15,472

Country	Data k	Data s_R	Data A	Model A
Quartile 1	2.0	0.4%	4,478	2,184
Quartile 2	2.5	0.5%	9,574	5,358
Quartile 3	3.1	1.7%	11,111	11,763
Quartile 4	2.9	1.7%	15,441	12,286

Table 8 Model A versus data A ($\theta = 0$ case)

A parameter remaining to calibrate is θ .²² Before discussing possible values for this parameter, it is useful to consider the case where $\theta = 0$ – so that there is no effect of distance on international spillovers – and to compare the implications of the model to the data. Using the R&D investment rate data of Lederman and Saenz (2003) and our estimated k levels, Equation (4.17) yields the model's implied relative A level for each country (a_i) . We want to compare this against the data. To do so, we use the value of A we calculated for the U.S. in the previous section and $a_{\rm US}=0.7$ to obtain an implied value for the world technology frontier, A^* (recall that with $\theta = 0$ there is a well defined technology frontier that is common to all countries). We can then obtain the model's implied A values for all countries using $A_i = a_i A^*$. The result of this exercise is shown in Table 8, where we divide countries into four groups according to their levels of A and show the median of the different variables for each group. It is clear that the model does badly for the poorest countries, predicting much lower A levels for them than occur in the data. This discrepancy does not occur for the richest countries, so the model is predicting significantly larger A differences than in the data. For example, whereas (according to the data) the top group's median A is 3.4 times the median A of the bottom group, the model implies a ratio of 5.6.

The model implies large differences in productivity in response to small differences in R&D investment rates. As is well known, the neoclassical model – with only around 1/3 share for physical capital – cannot generate large differences in steady state labor productivity in response to modest differences in investment rates [see the discussion in Lucas (1990)]. It is worth pausing here to explore some of the reasons behind these divergent properties. Manipulating the neoclassical model, one can show that the semi-elasticity of steady state labor productivity with respect to the investment rate is given

The higher value of λ , however, would imply a high relative A level for the U.S. and consequently – given measured A for the U.S. – a value for A^* that would be lower than the measured A levels of the high A countries, such as Hong Kong and Italy. To avoid this, we choose the lower value of λ .

²² We must also set a value for σ , which is crucial for determining the level of \tilde{A} . We use the value of $A_{\rm US}$ obtained from the data, (4.18)–(4.21), $a_{\rm US}=0.7$, and a value for $z_{\rm US}$ (equal to one when $\theta=0$ and a known value from the solution to the above system of equations for the case $\theta>0$) to arrive at a value for σ . In the case with $\theta=0$, then $z_{\rm US}=1$, and hence $\tilde{A}=A_{\rm US}/a_{\rm US}=22,196$. Given the value of aggregate L in our sample of countries, this implies from (4.20) that $\nu=0.00082$, which plugging into Equation (4.21) implies finally that $\sigma=0.0023$.

by:

$$\frac{\partial \ln y}{\partial s} = \frac{r}{1 - \alpha} \cdot \frac{1}{\delta + g_A + g_L}.$$
 (4.23)

With the values we used above ($\alpha=1/3$, $\delta=0.08$, $g_L=0.011$ and $g_A=0.015$), (4.23) yields a semi-elasticity of only 1.22% when evaluated at r=8.6%. Thus large differences in investment rates would be required to generate sizable differences in labor productivity across countries. Two differences between the way the R&D investment rate operates in our model and the way the physical capital investment rate operates in the neoclassical model stand out: first, the depreciation rate of ideas in our model is zero versus $\delta=0.08$ for capital in the neoclassical model; second, the elasticity of output with respect to the stock of ideas can exceed 1/3 (we have it at 2/3). To see the importance of these values, note that with $\alpha=2/3$ the semi-elasticity doubles to 2.46% (still with r=8.6%). If we use $\delta=0$ as well, then the semi-elasticity increases to 9.6%. In our model, the combined share of physical capital and ideas is actually 1. Without the constraint of the world technology frontier, therefore, the long run response of output would be infinite.

It is important to recall that the results shown in Table 8 and discussed above were derived for the case of $\theta=0$. Is it possible that a positive value of θ could improve the model's fit with the data? As will become clear below, countries with high levels of k and high R&D investment rates tend to cluster together. Thus, assuming a positive value for θ would actually make the model *less* consistent with the data, since it would imply an even larger difference between A levels across rich and poor countries.

One possible reason why the model is not doing well in matching the data is that measured R&D is not the appropriate empirical counterpart of "research" in the type of models we have been examining. In particular, measured R&D only includes formal research; this is research performed in an R&D department of a corporation or other institution. This fails to capture informal research, which may be particularly important in non-OECD countries. To explore this idea, in the rest of this section we assume that both R&D intensity and the productivity index A are measured with error. We estimate "true" R&D intensities by minimizing a loss function equal to the sum of two terms that capture, respectively, the deviation of the "true" R&D intensities from the data and the deviation of the model's implied (log of) A values from the data, with weights given by the standard deviation of the corresponding differences. In principle, we could follow this procedure for each value of θ . However, at $\theta = 0$, the partial derivative of our loss function with respect to θ is positive and large, implying that – just as argued above – the model's fit with the data worsens as θ increases from zero. Thus, we restrict ourselves

We do this in two stages. In the first stage we minimize a loss function without weights. We use the results to calculate the standard deviation of the error terms, or differences between data and "true" values for both R&D intensity and productivity. In the second stage we minimize the loss function with weights given by these calculated standard deviations.

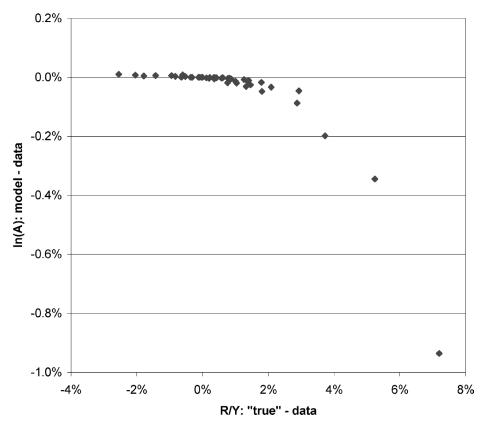


Figure 4. Deviations of the model from the data for research intensity and productivity.

to estimating R&D intensities for $\theta=0$ and later show what happens if, keeping the same R&D intensities estimated for $\theta=0$, we have positive values of θ .

It should be acknowledged that this procedure obviously implies that we can no longer evaluate the model's consistency with the data; our interest is now to explore the implications of the model for the differences in R&D investment rates that would be necessary to explain cross-country differences in A, as well as the implied differences in R&D tax rates that would be necessary to bring about those R&D investment rates.²⁴

The results of the exercise described above are shown in Figure 4 and Table 7 (columns 3 and 4). There are three points to note from these results. First, it is clear

²⁴ This exercise assumes that all differences in *A* across countries are causes by differences in technology adoption. An alternative view is that *A* differences arise from "distortions" that directly affect productivity through their impact on factor allocation, input usage, effort on the job, etc. We believe that understanding the relevance of barriers to technology adoption versus distortions is an important topic for future research.

that the procedure leads to only small deviations of A from the data, whereas the deviations are more significant for R&D intensities. It would appear that R&D intensities have more significant measurement problems (or are conceptually more different than research intensity in our model) than productivity levels. Indeed, the standard deviation of residuals of s_R with respect to the data is 0.12, whereas the corresponding value for the (log of) A is 0.01. Second, there are some countries for which the estimated R&D intensity is much higher than the data. Italy, for example, has a measured R&D intensity of 1.1%, whereas its "true" value is 8.3%. This arises because of Italy's high measured productivity (Italy's A is 24% higher than the U.S. level) and low value of k (2.6 versus 3.6 in the U.S.). Something similar happens for other high-A countries, such as Hong Kong and Ireland. Finally, just as one would expect given the results above, estimated R&D intensities vary much less than the corresponding values in the data. This is the main mechanism by which the procedure allows the model to fit perfectly. It also suggests that measurement error may be behind the low R&D intensities of several poor countries and of some high A countries such as Italy, Ireland and Hong Kong.

We can now explore what happens when θ is positive, so that spillovers decline with distance. Given the estimated R&D intensities, productivity levels change with θ only because of the associated changes in the variables z, which capture the effect of distance on spillovers for each country. In principle, we can obtain the values of $(z_1, z_2, \ldots, z_{48})$ for any $\theta \ge 0$ from the solution of a system of 48 non-linear equations represented by (4.22). Equation i of this system can be expressed as:

$$z_i^{(1-\gamma)+1} = \frac{\sum_{l=1}^{48} \lambda k_l s_{Rl} a_l z_l \omega_l e^{-\theta d(i,l)}}{\sum_{l=1}^{48} \lambda k_l s_{Rl} a_l \omega_l}.$$
 (4.24)

Solving this system numerically for the parameter values we have discussed and the R&D intensities derived before, we arrive at a value of z_i for each country, from which we can then obtain the country's level of A by using $A_i = a_i z_i \tilde{A}$ from (4.18) and (4.19).

What are reasonable values to use for the parameter θ ? Using industry level data on productivity and research spending across the G-5 countries, Keller (2002) estimated a reduced form model where cumulative industry research affects own productivity and also affects productivity in the same industry in other countries through international spillovers that decline with distance.²⁶ Given the similarity between Keller's system and a reduced form of our model, it seems reasonable to use Keller's estimate of θ , namely $\theta_K \equiv 0.0009$ in the calibration of our model. It turns out, however, that with $\theta = \theta_K$ our model cannot match the data – in particular, there is no solution to the system of Equations (4.24), at least for the parameters used for the exercises above.

 $^{^{25}}$ These standard deviations are the ones that arise after the two stage procedure described in the previous footnote. After the first stage, the standard deviations for the R&D rate and the (log of) A are 0.11 and 0.03, respectively.

²⁶ For other estimates of international spillovers from R&D, see Coe and Helpman (1995) and Coe, Helpman and Hoffmaister (1997). For a study of agricultural R&D spillovers, see Evenson and Gollin (2003). Becker, Philipson and Soares (2003) present evidence consistent with international spillovers of health technology.

			"T	rue"			a for
Country	$ au_K$	k	s_R	a	SRR	$ au_R$	$\tau_{Ri} = \tau_{R,\mathrm{US}}$
Quartile 1	13%	2.0	0.60%	20%	42%	102%	58%
Quartile 2	0%	2.5	1.13%	43%	37%	93%	68%
Quartile 3	4%	3.1	1.97%	50%	29%	31%	72%
Quartile 4	6%	2.9	2.98%	70%	21%	-16%	70%

Table 9
Implied R&D tax rates

Notes: τ_R is calculated as the level of τ_R needed to generate the "true" research intensity. For each country, we use its own implied income tax level (τ_K) and its own capital intensity level k. The last column presents the equilibrium steady state relative A level (a) for the hypothetical case in which all countries have the same R&D tax as the U.S. $(\tau_{Ri} = \tau_{R,US})$ but have different income tax rates and capital intensity levels.

This is because θ_K is unreasonably high. One way to see this is by noting that it implies a half distance of 746 miles: this implies that spillovers from the U.S. to Japan would be only one tenth of those to Mexico, and spillovers from the U.S. to New Zealand would be only one fifth of those to Japan.

We were able to find solutions for the system with $\theta = \theta_K/5$. For comparison, we also obtained solutions for two other values of θ , namely $\theta = \theta_K/10$ and $\theta = \theta_K/100$. A group of European countries (Belgium, France, United Kingdom, Germany, Ireland, Italy, and Netherlands) always come out with the highest values of z, whereas New Zealand always comes out with the lowest value. For $\theta = \theta_K/100$, $\theta = \theta_K/10$ and $\theta = \theta_K/5$, the minimum and maximum values of z are (93%, 96%), (48%, 68%) and (24%, 50%), respectively. Clearly, for high values of θ , geography by itself can lead to large differences in productivity across countries.

In the rest of this section, we focus on the case $\theta=0$, since – as explained above – the model's fit with the data is best at this point. (Recall that the model fits perfectly because we are using the estimated research intensities and the implied A values). Table 9 presents summary statistics for the solution for the case of $\theta=0$. Our discussion of these results will focus on the comparison of the poorest and richest quartiles (ordered, as above, in terms of A levels) in this table.

There are several points that we want to highlight in relation to these results. First, the median income tax is 13% and 6% for the poorest and richest countries, respectively. Everything else equal, this would lead to a lower R&D investment rate in the poorest countries. Second, as expected, rich countries have a higher k than poor countries: the level of k in these two groups is 2 and 2.9, respectively. As commented in Section 4.2, higher k has a direct effect on relative k (see Equation (4.17)) and an indirect effect (it could be positive or negative) through its impact on R&D investment rates (see Equation (4.16)). A natural question arises: is it the case that once we take into account the effect of k on TFP we can resuscitate the "neoclassical revolution" mantra that differences in physical and human capital accumulation rates account for most of cross-country income differences? More concretely, how much of the variation in k

levels across countries is due to the variation in levels of k? A simple way to answer this question is to note from Equation (4.17) that differences in relative A levels are driven by differences in the product $s_R k$ across countries. Running a regression of s_R on the log of this product yields a coefficient of 0.8, which implies that when $s_R k$ increases by one percent, we should expect s_R to increase by 0.8%. Clearly, most of the variance of the product $s_R k$ is accounted for by the variance of s_R .

Third, the social return to R&D is higher for poor countries. This is consistent with the findings in Lederman and Maloney (2003) and also with the idea that poor countries have policies and institutions that negatively affect the quantity of research.

Fourth, the column with heading τ_R indicates the R&D tax rate required to produce the "true" R&D investment rates given each country's levels of τ_K . The main question we address here is whether differences in income tax rates, which affect both the rate of investment in physical capital and R&D, are sufficient to explain differences in estimated research intensities. The answer is clearly negative: the required R&D tax rate in the poorest countries is 102% compared to -16% in the richest countries. To address the same question from a different angle, the last column calculates each country's implied relative A level if all countries had the same R&D tax as the U.S. but kept their own levels of τ_K . It is clear that differences in τ_K alone are too small to account for the wide dispersion in productivity levels across countries.

Finally, as emphasized above, the results in Table 9 suggest that small differences in steady-state R&D investment rates have large effects on steady state relative A levels. For example, in the calibrated model, by increasing its R&D investment rate by 1% from 0.6% India could double its steady state relative A level from 17% to 34%, clearly a very large effect. India's social rate of return to research, however, is a moderate 30%. The apparent contradiction arises because the large effect of the increase in the R&D intensity on the relative A level is a steady-state comparative-statics result, and hence does not take into account the transition, which is a crucial component in the calculation of the social rate of return to R&D. As a result, in spite of the large effect of differences in R&D investment rates on relative A levels in steady state, the required implicit taxes on R&D are not huge.

4.5. The benefits of engagement

One of the benefits of the model we have constructed is that it allows us to perform an interesting exercise. We can ask: how much do countries benefit from spillovers from the rest of the world?²⁷

First, note that a country's equilibrium a_i is not affected by being isolated or engaged. Thus, the whole benefit of engagement is going to captured by the way engagement affects the term z_i . Now, if a country is isolated, or disengaged, its equilibrium z would be characterized by the solution to the system (4.24) when $\theta \to \infty$. It is easy to check

²⁷ See Eaton and Kortum (1999) for a host of related counterfactuals with similar outcomes.

Country	Share of world's L	Scale effect	S.V. effect	Total effect
U.S.	7.1%	37	0.12	5
UK	1.5%	297	0.21	64
Belgium	0.2%	4,093	0.12	480
Brazil	3.1%	114	0.97	110
India	1.3%	9	23.0	217
China	38.7%	4	70.6	258
Senegal	0.2%	4,451	42.0	187,035

Table 10 Benefits of engagement for selected countries

that this yields

$$\tilde{z}_i = \left(\frac{\lambda k_i s_{Ri} a_i \omega_i}{\sum_l \lambda k_l s_{Rl} a_l \omega_l}\right)^{1/(1-\gamma)}.$$
(4.25)

Thus, the benefits of engagement are captured by z_i/\tilde{z}_i . From (4.17) we get

$$z_i/\tilde{z}_i = z_i \left(\frac{\sum_l \omega_l v_l}{\omega_i v_i}\right)^{1/(1-\gamma)}$$

where $v_i \equiv a_i \lambda k_i s_{Ri} = \lambda R_i / A_i^* L_i$ is a measure of research intensity. Letting $\bar{v} \equiv \sum_i \omega_j v_j$ be the world's weighted average of v_i , we obtain

$$\frac{z_i}{\check{z}_i} = z_i \left(\frac{1}{\omega_i}\right)^{1/(1-\gamma)} \left(\frac{\bar{\upsilon}}{\upsilon_i}\right)^{1/(1-\gamma)}.$$
(4.26)

The first term on the RHS of this equation, z_i , captures the fact that even when fully engaged, a country's technology frontier is inferior to the world's frictionless frontier if $\theta > 0$, in which case $z_i < 1$ for all i. The second term is the pure scale effect that arises in this model. The third term, which we call the "Silicon Valley" effect, captures the fact that richer countries benefit less from being part of the world than poor countries do because of their higher effective research intensity.

Table 10 presents results based on these values and assuming $\theta = 0$, which implies $z_i = 1$ for all i. The results suggest huge benefits of engagement. At the extreme, Senegal's productivity is 187 thousand times higher than it would be if it was isolated! Of course, if $\theta > 0$ then $z_i < 1$ and the overall effect would be small. Still, it is our conjecture that any reasonable value of θ would still imply enormous benefits of engagement. Of course, in a more general model, it is reasonable to think that productivity could not fall below a certain level because of Malthusian forces. Specifically, suppose there is a fixed factor such as land. Then, for sufficiently low A, population would decline until income per capita was equal to the subsistence level. Instead of very low levels of A, disengagement would mean very low population sizes. Put differently, an important part of the benefits of engagement may be realized through larger population rather than

higher productivity. The implications are clear: if it were not for the benefits of sharing knowledge internationally, countries would have much lower productivity levels and populations than they now do.

4.6. Discussion of main results

We finish this section with a discussion of the main results we want to emphasize.

First, the usual separation between capital and productivity – or between investment and technological change – is not always valid. For a given R&D investment rate, higher investment rates in physical and human capital lead naturally to higher TFP productivity levels. Thus, one should not jump from cross-country dispersion in TFP to the conclusion that differences in physical and human capital play a minority role in accounting for international income differences. When we calibrate our model, however, we find that differences in R&D investment rates account for most of the cross country variation in productivity.

Second, international variation in R&D investment appears *more* than large enough to generate the international variation in productivity. But it seems likely that measured R&D does not capture all of the investment associated with adoption of foreign technology. Indeed, we find that countries such as Indonesia, Peru and Senegal have R&D investment rates that are much too low to be consistent with their productivity levels. It is likely that their true research intensities are much higher than the measured ones. We hope to see more research in understanding how to capture and measure "research".

Third, differences in (implicit) capital income tax rates are not large enough to account for the observed differences in R&D investment rates and productivity levels. The calibrated model suggests that sizable differences in R&D taxes are needed. These R&D taxes are clearly not formal or explicit taxes, but the result of policies and institutions that make research more costly or reduce its associated returns. Exploring the nature and source of these differences in implicit R&D taxes across countries is an important topic for future research.

Finally and most importantly, the calibrated model indicates that countries benefit enormously from international knowledge spillovers. We think any reasonable value of θ (which governs the rate at which spillovers decline with distance) would yield results similar to those we presented above.

5. Conclusion

Externalities are not theoretically necessary to sustain growth. But they appear essential for understanding why many countries grow at similar rates despite differing investment rates. A dramatic way to summarize the importance of international knowledge externalities is to calculate world GDP in the absence of such externalities. According to our calibrated model, world GDP would be only 6% of its current level, or on the order of \$3 trillion rather than \$50 trillion, if countries did not share ideas. Such scale effects

from the nonrivalry of knowledge are a central theme in the works of Romer (1990), Kremer (1993), Diamond (1997), Jones (2001, 2005) and many others.

Because diffusion is not costless, however, differences in knowledge investments may explain a significant portion of income differences across countries. We show that modest barriers to technology adoption could account for differences in TFP of a factor of four or more, as observed in the data. But we have not documented such barriers to knowledge diffusion in practice. We consider this a priority for future research. A related issue is that the high power of the model in explaining large productivity differences from small barriers to technology adoption comes at a cost, namely that the transition from one steady state to another after a discrete change in barriers takes a long time. This is what allows small changes in barriers to have large steady-state effects without implying huge social rates of return to research in poor countries. But it implies that the model is not very useful in understanding miracles. Perhaps a different formulation of the "benefits of backwardness" in Equation (4.2) would allow for a more attractive balancing of steady state and transition properties.

We have also left for future research the identification of the primary channels of international knowledge spillovers. Trade, joint ventures, FDI, migration of key personnel, and imitation may all play important roles. See Keller (2004) for a survey of recent empirical work on this topic. A model with trade would lead naturally to some countries having a comparative advantage in doing innovative R&D and other countries focusing on adoption and imitation R&D. The evidence on international patenting supports the notion that innovative R&D is concentrated in rich countries. Of course, countries can imitate other imitators as well as the original innovators. We hope to see future research documenting not only the vehicles for knowledge diffusion, but their specific routes.

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Appendix A

The firm's maximization problem can be restated as choosing \dot{A}_{jis} and \dot{K}_{jis} to maximize:

$$\int_{t}^{\infty} \left((1 - \tau_{Ki}) K_{jis}^{\alpha} (A_{jis} h_{i})^{1-\alpha} - p_{i} \dot{K}_{jis} - p_{i} \delta K_{jis} - \frac{(1 + \tau_{Ri})}{(1 - \mu)\lambda} \left(\frac{\dot{A}_{jis}}{1 - A_{jis}/A_{is}^{*}} - \varepsilon A_{jis} - \mu \lambda \overline{R}_{is} \right) \right) e^{-r(s-t)} ds.$$

Letting Q represent the expression in the integral, then we know that a solution to this problem must satisfy the following Euler equations: $\partial Q/\partial K_{jis} = \frac{d}{ds}(\partial Q/\partial \dot{K}_{jis})$ and

 $\partial Q/\partial A_{jis} = \frac{d}{ds}(\partial Q/\partial \dot{A}_{jis})$. The first Euler equation is:

$$\frac{\alpha Y_{jis}}{p_i K_{jis}} = \frac{r + \delta}{1 - \tau_{Ki}}.$$

Since in a symmetric equilibrium the capital—output ratio of firm j is the same as the aggregate capital output ratio, then this implies that:

$$\frac{p_i K_{it}}{Y_{it}} = \alpha \frac{1 - \tau_{Ki}}{r + \delta}.$$

As to the second Euler equation, differentiation yields (we are using the symmetry condition for the equilibrium):

$$\frac{\partial Q}{\partial A_{jis}} = \left((1 - \tau_{Ki})(1 - \alpha)Y_{jis}/A_{jis} - \frac{(1 + \tau_{Ki})}{(1 - \mu)\lambda} \left(\frac{g_A a_i}{(1 - a_i)^2} - \varepsilon \right) \right) e^{-r(s-t)}$$

and

$$\frac{\partial Q}{\partial \dot{A}_{iis}} = -\frac{(1+\tau_{Ri})}{(1-\mu)\lambda(1-a_i)} e^{-r(s-t)}.$$

Hence.

$$\frac{\mathrm{d}}{\mathrm{d}s} \left(\frac{\partial Q}{\partial \dot{A}_{iis}} \right) = \frac{r(1 + \tau_{Ri})}{(1 - \mu)\lambda(1 - a_i)} \mathrm{e}^{-r(s - t)}.$$

Thus, the Euler equation is:

$$\frac{(1 - \tau_{Ki})(1 - \alpha)Y_{jis}}{A_{jis}} - \frac{(1 + \tau_{Ri})}{(1 - \mu)\lambda} \left(\frac{g_A a_i}{(1 - a_i)^2} - \varepsilon \right) = \frac{r(1 + \tau_{Ri})}{(1 - \mu)\lambda(1 - a_i)}.$$

Noting that in a symmetric equilibrium we must have $Y_{jis}/A_{jis} = Y_{is}/A_{is}L_{is} = k_i$, and manipulating, we get:

$$\Omega_i(1-\alpha)\lambda k_i(1-a_i) - \frac{g_A a_i}{1-a_i} + \varepsilon(1-a_i) = r$$

where

$$\Omega_i \equiv \frac{(1 - \tau_{Ki})(1 - \mu)}{(1 + \tau_{Ri})}.$$

Comparative statics

(From here onwards we drop the subscripts.) It is easy to show that a is increasing in both Ω and k. In particular:

$$\frac{\partial a}{\partial k} = \frac{\Omega(1-\alpha)\lambda(1-a)}{\Omega(1-\alpha)\lambda k + (g_A/(1-a)^2) + \varepsilon} > 0.$$

Differentiating $g_A = (\lambda ks + \varepsilon)(1 - a)$ (using s for s_R) we get

$$(\lambda s \, dk + \lambda k \, ds)(1 - a) - da(\lambda ks + \varepsilon) = 0.$$

This implies that:

$$\lambda k \frac{\partial s}{\partial k} = \frac{(\partial a/\partial k)(\lambda ks + \varepsilon)}{(1-a)} - \lambda s.$$

Plugging in from the result above we finally get:

$$k\frac{\partial s}{\partial k} = \frac{\Omega(1-\alpha)(\lambda ks + \varepsilon)}{\Omega(1-\alpha)\lambda k + (g_A/(1-a)^2) + \varepsilon} - s.$$

Summing on the RHS and noting that the denominator is clearly positive we get that $\partial s/\partial k > 0$ if and only if:

$$\Omega(1-\alpha)\varepsilon - s\frac{g_A}{(1-a)^2} - \varepsilon s.$$

This term could well be negative.

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