Entry Costs Rise with Development

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Abstract
Across cohorts of plants (or firms) within the U.S., India and China, we find that average discounted profits rise systematically with average labor productivity at the time of entry. In models with a zero profit condition for entrants, these facts imply that the cost of creating a new business increases as development proceeds. This force dampens the welfare impact of policies in a host of models of firm dynamics, growth, and trade.

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1. Introduction

As countries develop, do they create more firms or just better ones? Suppose that new businesses are created with a fixed amount of output. Then a policy which boosts productivity can generate an endogenous expansion in the number of firms, which can increase variety or reduce span-of-control costs. This multiplier effect through entry is analogous to the multiplier effect on output from physical capital accumulation in the neoclassical growth model. If instead entry requires a fixed amount of labor, then policies boosting productivity are not amplified through entry because entry costs rise with the price of labor.

Widely used models of firm dynamics, growth, and trade make different assumptions about entry costs. Some models assume entry costs are stable (e.g. a fixed output cost to invent a new product). Other models assume entry costs rise with as growth proceeds, say because entry requires a fixed amount of labor and labor becomes more expensive with growth. Some studies do not take a stand but emphasize that the entry technology matters for the welfare impact of policies.

Existing evidence is limited on how entry costs change with growth and development. This is why models are mixed or agnostic on the question. The evidence is mostly confined to estimates of the regulatory barriers to entry across countries, to the exclusion of the technological costs of innovating and setting up operations. Djankov et al. (2002) document higher statutory costs of entry (relative to GDP per capita) in poor countries. Their pioneering effort spawned the influential Doing Business surveys conducted by the World Bank.

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1By “entry costs” we have in mind all non-production costs over a firm’s life cycle. These include not only upfront innovation and setup costs but also overhead costs, R&D of incumbents, and fixed costs of exporting.


4See Rivera-Batiz and Romer (1991), Atkeson and Burstein (2010), Bhattacharya et al. (2012), the survey by Costinot and Rodriguez-Clare (2013), and Atkeson and Burstein (2015).
ENTRY COSTS RISE WITH DEVELOPMENT

The overall distribution of employment across firms and plants provides some indirect evidence. Laincz and Peretto (2006) report no trend in average firm employment in the U.S. Luttmer (2007, 2010) shows that entry costs proportional to average productivity are necessary for the existence of a stationary firm size distribution in various growth models. Across countries, however, Bento and Restuccia (2015) document higher employment per establishment in richer countries.

In this paper, we use the zero-profit-condition to infer entry costs from profits after entry. We measure expected profits after entry in three ways: 1) the lifetime PDV of profits per firm or plant in a cohort; 2) profits per entering firm or plant at the time of entry; and 3) average profits per firm or plant across all firms. We look at how these measures vary with the level of labor productivity over time in the U.S., India and China. Measure 1) is the ideal but requires tracking many cohorts over their lifetimes. We use this measure whenever data allows. Measure 2) is equivalent to Measure 1) when there are constant markups and rates of post-entry growth, exit, and discounting. We calculate this measure for all of our countries and check the validity of the assumptions whenever data is available. Measure 3) is a special case of Measure 2) where the post-entry growth is not only constant but zero.

We find that all three of our proxies for expected profits increase strongly with average labor productivity in the economy. Meanwhile, the number of firms and establishments is closely tied to aggregate employment over time. We show that these simple empirical elasticities discipline the nature of entry costs in widely used models. In particular, if a zero-profit-condition holds, then entry costs must be rising with average labor productivity in the economy or cohort at the time of entry.

We illustrate the implications of our empirical findings for modeling and policy in a stylized Melitz model. In this model, entry costs could rise with

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5For the U.S. we use the 1963–2012 quinquennial Census of Manufacturing microdata. For India we rely on the 1989–2007 Annual Survey of Industries. For China we analyze the 1998–2007 Surveys of Industrial Production.
the level of development simply because entry is labor-intensive and labor becomes more expensive when labor productivity grows. Entry costs could also rise with development because entrants set up more technologically sophisticated operations as the economy advances.\footnote{Our evidence is relevant for total entry costs, i.e. the sum of technological and regulatory barriers. If, as seen in the Doing Business surveys, regulatory entry costs increase modestly or even fall with development, then technological entry costs must be the dominant force pushing up entry costs with development.} We use our empirical findings to estimate parameters in the model which govern the labor-intensity of entry costs and the relationship between entry costs and the level of technology. We find that fitting our facts requires that entry is labor-intensive and/or that better technology calls for higher entry costs.

We draw the following three tentative conclusions for modeling and policy. First, if the choice is between fixed entry costs in terms of labor or output, our evidence favors denominated entry costs in terms of labor. Second, our evidence is consistent with the assumption of rising innovation costs with technological progress, as is often assumed to obtain balanced growth in theory.\footnote{E.g. Romer (1990), Aghion and Howitt (1992), Kortum (1997), and chapters 13 and 14 of Acemoglu (2011).} Third, productivity-enhancing policies appear to have muted effects on entry.

The rest of the paper proceeds as follows. Section 2 quickly describes how entry costs relate to profits under a zero-profit-condition. Section 3 presents evidence on how profits per firm or plant increase as development proceeds. Section 4 presents a few simple models to illustrate why we care about the nature of entry costs and how to use our evidence to discipline the nature of entry costs. Section 5 concludes.

## 2. Entry costs and the zero-profit-condition

In this paper, we use the present discounted value (PDV) of profits to estimate the cost of creating a firm. This approach is consistent with many workhorse
models of firm dynamics used in macroeconomics and trade. In these models, the equilibrium definition includes a zero-profit-condition that requires the cost of entering to be less than or equal to the value of entering, and to be strictly equal when entry is positive. Let $c_e(t)$ denote the cost of entry for a firm in units of output at time $t$. Let $M_e(t)$ denote the number of entrants and $E_t V(t)$ the expected value of entering. The zero-profit-condition assumes

$$M_e(t) \left( E_t V(t) - c_e(t) \right) = 0, \quad c_e(t) \leq E_t V(t), \quad M_e(t) \geq 0.$$  

In the case of a risk-neutral firm, the value of entering is simply equal to the PDV of profits:

$$E_t V(t) = E_t \sum_{\text{age}=0}^{\infty} \lambda(t, \text{age}) \pi(t, \text{age})$$  

where $\lambda(t, \text{age})$ denotes the real discount rate. We assume a representative household owns all of the firms and hence all firms discount profits by the same rate. $\pi(t, \text{age})$ denotes the flow of profits in units of current output.

The zero-profit-condition and equation (1) imply that entry costs are equal to the expected PDV of profits when there is positive entry ($M_e(t) > 0$). Hence, according to models with a zero-profit-condition, we can infer how entry costs move with the level of average productivity by analyzing how the expected PDV of profits trends with average productivity.

We measure the expected PDV of profits in three ways. When data is available, we simply measure it by the realized PDV of profits per firm or plant for a cohort. So our first proxy for entry costs in period $t$ is

$$\frac{1}{M_e(t)} \sum_{f=1}^{D_f} \sum_{\text{age}=0}^{\lambda(t, \text{age}) \pi_f(t, \text{age})}$$  

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9We take the behind-the-veil approach of Hopenhayn, whereby all entrants draw from the same (say productivity) distribution but do not know their realization before entering. In the Appendix, we consider the case of the zero-profit-condition holding for a marginal entrant because entrants know what their profits will be ex ante.
10In many models firms receive profits because of market power and/or fixed factors.
where $f$ indexes the firm and $D_f$ denotes the age of the firm at exit (death).

Implementing (2) requires tracking cohorts over their lifetime. We currently have reasonable data for this task for the U.S., and are building toward it for India. To analyze countries that do not have such data, we use a second measure of the expected PDV of profits. Namely, we assume constant rates of post-entry growth, exit, price-cost markups and discounting. Under these conditions, the PDV of profits is proportional to the average profits for entrants at entry:

$$\frac{1}{M_e(t)} \sum_{f=1}^{M_e(t)} \pi_{f}(t, age = 0)$$

(3)

We construct a third measure of the expected PDV of profits from the average profit across all firms operating in period $t$:

$$\frac{1}{M_t} \sum_{f=1}^{M_t} \pi_{f}(t)$$

where $M_t$ is the total number of firms. This third measure is a special case of the second measure in (3) wherein firms operate for only one year, or in which entrant profits are proportional to average profits.

For each of the three measures we must estimate the flow of profits. Rather than trying to distinguish economic and accounting profits or variable and fixed costs, we estimate price-cost markups and combine our estimates with revenue to infer profits. Now, estimating the *level* of markups is notoriously difficult. Fortunately, for our purposes we only need to know how markups vary over time. We follow Bils et al. (2015) in using the inverse ratio of shipments to intermediates. Intermediates have the advantage of being a variable input less subject to adjustment and overhead costs than capital or labor. See the Appendix for details.
3. Empirical Patterns

3.1. The U.S.

3.1.1. Measure 1: lifetime present discounted value

We use establishment-level data in the Census of Manufacturing (CMF) by the U.S. Census Bureau from 1963 and quinquennially 1972 to 2012. The CMF covers all establishments with employees. For our sample period, there are about 1.54 million unique establishments.

Since the data covers all employer establishments, we construct cohorts by the first year the establishment appears in the data. This means that we drop all observations in 1963, because we cannot identify when these plants entered; we use the 1963 plants to determine which of the 1967 plants are entrants. We also drop 7.6K plants that exit and then re-enter, as their entry year is ambiguous. We drop all plant-years with negative or missing shipments and/or employment.

We calculate the PDV of profits for each cohort in the following way. First, we multiply shipments by the profit share (implied by our time-varying markup estimates) to generate profits for each plant-year. We deflate all profits by the BEA manufacturing value added deflator.\footnote{Since we only have data every five years, for each plant we interpolate real profit between years to generate yearly profits. We linearly interpolate the log of real profits, which is equivalent to fitting a constant growth rate of real profits between adjacent observations.} We discount each year of real profits assuming a constant real interest rate \( r = 0.05 \). We calculate the PDV of real profits for each cohort using horizons of 5, 10 and 15 years. A shorter horizon gives us more observations. The PDV for each cohort should be an unbiased estimate of its entry cost, given a zero expected profit condition for entrants.

We use real value added per worker each year to proxy for the level of development. We deflate total value added per worker in each year by the BEA manufacturing value added deflator. We calculate the total value added and total number of workers by summing value added and employment across plants.
Across entering cohorts, we regress the log of the PDV of real profits on the log of real value added per worker in the year of the cohort’s entry. Table 1 presents the results. At the 5 and 10 year horizons the PDV of profits rises even more than one-for-one with labor productivity at the time of entry (a slope above 1). The standard errors are small (.05 or less) and the $R^2$’s are large (0.8 or higher). At the 15-year horizon the PDV of profits increases less than one-for-one with labor productivity at entry, but the connection is still quite positive (slope 0.57). Thus, at all horizons, it appears that entry costs rise strongly with average labor productivity in U.S. manufacturing.

— Figures to be added once Census Bureau approves disclosure. —

### Table 1: PDV of profits on value added per worker, U.S. manufacturing

<table>
<thead>
<tr>
<th>Horizon in years</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient on $Y/L$</td>
<td>1.133</td>
<td>1.108</td>
<td>0.574</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.051)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.864</td>
<td>0.799</td>
<td>0.813</td>
</tr>
<tr>
<td>First cohort</td>
<td>1967</td>
<td>1967</td>
<td>1967</td>
</tr>
<tr>
<td>Last cohort</td>
<td>2007</td>
<td>2002</td>
<td>1997</td>
</tr>
<tr>
<td># of cohorts</td>
<td>9</td>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>
We find that real average profits of entrants increased strongly with real value added per worker: the elasticity is 1.198 (s.e. 0.008) across the 10 cohorts from 1967, 1972, ..., 2012 in the U.S. Census of Manufacturing.

Average profits of entrants behave similarly to the PDV of profits for entrants because markups, exit rates, and the growth rate of shipments for survivors exhibit little variation over time relative to the strong upward trends in shipments per plant and in value added per worker across cohorts.

— Figures to be added once Census Bureau approves disclosure. —

3.1.3. Measure 3: average profits of all firms

Whereas above we focused on entrants, one can also calculate average profits for all establishments. The benefit of doing so is that some publicly available datasets contain such data on all establishments but not entrants separately.

We find that average real profits likewise increase strongly with real value added per worker over time. The elasticity is 1.034 (s.e. 0.019) for the 10 years from 1967 through 2012.

3.2. Alternative measures of profits

The aforementioned results might be sensitive to using the inverse intermediate share to estimate movements in the price-cost markup. If the markup is instead fixed over time, then one can use shipments as a proxy for profits. When we regress real gross output per entering plant on real value added per worker at the time of entry, we obtain an elasticity of 0.841 (s.e. 0.038). We can also look at real gross output per worker for all establishments, not just entering ones. We find a similar elasticity of 0.815 (s.e. 0.005).

Another robustness check is to simply compare trends in value added per plant and value added per worker — both for all plants. This approach requires only a few summary statistics that are often available publicly, even if one does not have access to micro data. This is true for U.S. manufacturing. Moroever,
we can extend the time frame back to 1947. Figure 1 shows the Census years for all of manufacturing. Value added is again deflated by the BEA’s manufacturing value added deflator. As shown, value added per plant increases strongly with value added per worker. As real labor productivity grew by a factor of 5.6 in U.S. manufacturing from 1947 to 2012, value added per plant grew by a factor of 3.3 — implying an elasticity of around 0.7.

All of the U.S. results are consistent with the hypothesis that average plant size and profits increase reliably with average plant productivity. This evidence is consistent with entry costs trending up with labor productivity.

**Figure 1: Value added per plant vs. value added per worker, U.S.**
3.3. India

For India, we have establishment-level data from the Annual Survey of Industries (ASI) made available by the government’s Central Statistical Organization from 1985 to 2007. The ASI covers all registered manufacturing plants with more than fifty workers (one hundred if without power) and a random one-third sample of registered plants with between ten and fifty workers (twenty and 100 if without power). We use ASI-provided sampling weights in all our calculations. After cleaning, the raw data contains about 55,000 establishments per year. As for the U.S., we use data on shipments, intermediates, and employment.

3.3.1. Measure 1: lifetime present discounted value

— In progress. We are currently extending the sample period to 2011. —

3.3.2. Measure 2: average profits of entrants

Figure 2 displays the ratio of gross output to the cost of intermediate goods each year. The ratio exhibits little trend. This is consistent with stable increasing price-cost markups over time.

Figure 3 plots real value added per entering firms in a year against real value added per employee across all firms. Value added is measured by the difference between gross output and intermediate inputs. We deflated this measure by the World Bank manufacturing value-added deflator to create a measure of real value added. We find that 1 ppt increase in labor productivity is associated with 1.6 ppt increase in average entrant revenue. Taken together with the pattern of cost shares, these facts suggest that the average profits of entrants are increasing over time. Under the zero-profit-condition, rising profits suggest that entry costs increased over time with labor productivity.
Figure 2: Gross output relative to intermediate spending, India

3.3.3. Measure 3: average profits of all firms

Figures 4 shows that average value added per plant in India increased with manufacturing value added per worker with an elasticity of around 0.9. As real labor productivity grew by a factor of 4 in Indian manufacturing from 1985 to 2007, value added per plant grew by a factor of 3.3.

As in the U.S., in India plant size and profits trend up consistently along with plant labor productivity, consistent with the hypothesis that entry costs rise as development proceeds.

3.4. China

For China, we have firm-level data from the Survey of Industrial Production for 1998 to 2007.\textsuperscript{12} The survey is conducted by the National Bureau of Statistics.

\textsuperscript{12}The U.S. and Indian data was for plants. For the U.S. we can look at firms as well, and intend to do so. For both the U.S. and India, we can calculate the fraction of single-plant firms as well.
Figure 3: Value added per entering establishment vs. per worker, India

Figure 4: Value added per establishment vs. value added per worker, India
It covers all non-state-owned firms with more than 5 million yuan in revenue plus all state-owned firms. The raw data contains about 165K firms in 1998 and grows to around 340K firms in 2007.

We use shipments, intermediates, and production workers for each firm. We compute real value added using the World Bank manufacturing value added deflator for China. We restrict the sample to private firms, dropping state-owned enterprises because the free entry condition is debatable for them (to put it mildly). We think China is worth examining despite the short sample of available years because its manufacturing growth was so rapid.

3.4.1. Measure 3: average profits of all firms

Figure 5 shows the average value added per firm in China increased with manufacturing value added per worker with an elasticity of around 0.7. As real labor productivity grew by a factor of 2.7 in Chinese manufacturing from 1998 to 2007, value added per firm grew by a factor of 2.2.

As in the U.S. and India, in China business size and profits grew quickly along with business value added per worker, suggesting entry costs grew as well.

4. Illustrative Models

We now use several models to illustrate how entry costs rising with development can matter for welfare.

4.1. Love-of-Variety

First consider a static, closed economy version of the Melitz (2003) model. The economy has a representative household endowed with $L$ units of labor. Con-

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13 The results are robust to including the SOE firms.

14 Measure 3 in all three countries is robust to looking at industry-specific versions and controlling for industry fixed effects. Our results are not an artefact of compositional changes.
assumption per capita, which is proportional to the real wage $w$, is a measure of welfare in the economy.

Consumption goods are produced by a perfectly competitive sector that uses intermediate goods as inputs and a CES production technology. Profit maximization yields a downward sloping demand curve for each intermediate good.

The intermediate goods sector is monopolistically competitive. Without loss of generality, we assume all firms in this sector have the same production function, which is linear in labor inputs with technology level $A_y$. Each intermediate goods firm takes demand for its product as given and chooses its output or price to maximize its profit. This yields the familiar relationship between the wage bill, revenue, and profit in each firm

$$\begin{align*}
wl &= \frac{\sigma - 1}{\sigma} py \\
&= (\sigma - 1) \pi
\end{align*}$$

where $Y$ is aggregate output and $\sigma > 1$ is the elasticity of substitution between

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15We could allow post-entry heterogeneity in firm technology and define $A_y := (E A_{y^1})^{\frac{1}{1 - \tau}}$. 
varieties. Let $L_y$ be the total amount of labor devoted to producing intermediate goods and $M$ the total number of intermediate goods produced. By symmetry of the intermediate goods production function

$$Y = A_y L_y M^{\frac{1}{\sigma-1}}. \tag{5}$$

One unit of an entry good is required to create a variety, i.e., set up an intermediate goods firm. We generalize the production technology of the entry good in Melitz (2003) to allow final goods to be an input into creating a new variety. In particular, we follow Atkeson and Burstein (2010, 2015) in assuming that the entry technology has the Cobb-Douglas form

$$M = A_e Y_e^{1-\lambda} L_e^\lambda \tag{6}$$

where $L_e$ and $Y_e$ are the amount of labor and final output, respectively, used in creating varieties.

Perfect competition in the CRS sector producing entry goods implies that the cost of creating a variety in terms of consumption goods is

$$c_e \propto w^\lambda A_e. \tag{7}$$

And the labor share of revenue in entry goods production is

$$w L_e = \lambda c_e M. \tag{8}$$

Free entry, with positive entry in equilibrium, implies

$$\pi = c_e \tag{9}$$

which equates profit per variety to the entry cost.

Thus, the one-shot equilibrium given $(L, A_y, A_e)$ consists of prices $(w, c_e)$ and allocations $(C, M, Y, L_e, L_y)$ such that (4) to (9) hold, and the following labor and
goods market clearing conditions are satisfied:

\[ L = L_y + L_e, \quad Y = C + Y_e. \]

We now consider how the welfare impact of a change in \( A_y \) depends on the entry technology. In equilibrium, welfare (the real wage) is

\[ w = \frac{\sigma - 1}{\sigma} A_y M^{\frac{1}{\sigma - 1}} \]

so

\[ \frac{\partial \ln w}{\partial \ln A_y} = 1 + \frac{1}{\sigma - 1} \frac{\partial \ln M}{\partial \ln A_y}. \]

An increase in \( A_y \) not only raises welfare directly, but also has the potential to improve welfare indirectly through variety expansion.

One can show that equilibrium variety satisfies

\[ M \propto \frac{wL}{c_e} \]

so that the number of varieties depends on the value of labor relative to the entry cost. Combining this with equation (7) relating the real wage to \( c_e \), we get

\[ \frac{\partial \ln M}{\partial \ln A_y} = (1 - \lambda) \frac{\partial \ln w}{\partial \ln A_y} \]

That is, the elasticity of variety with respect to \( A_y \) is larger when the share of output used in producing varieties \((1 - \lambda)\) is bigger. Higher \( A_y \) means more output, and some of this output is devoted to producing more varieties if the final good is used in entry \((\lambda < 1)\). Repeated substitution will show that the compounding impact of \( A_y \) on welfare is

\[ \frac{\partial \ln w}{\partial \ln A_y} = 1 + \frac{(1 - \lambda)}{\sigma - 1 - (1 - \lambda)} \]
with the second term capturing the effect of variety expansion. A higher output share \((1 - \lambda)\) means more amplification.

The amplification of an increase in productivity depends on \(\sigma\), the degree of substitutability of intermediate goods, because varieties are more valuable when substitutability is low. To illustrate the potential importance of variety expansion, consider the Broda and Weinstein (2006) estimates of \(\sigma \approx 4\) at the 3-digit to 4-digit product level. For this value of \(\sigma\), the amplification (ratio of amplified impact to direct impact) can range from 50% when \(\lambda = 0\) to 0% when \(\lambda = 1\). Thus, for a plausible value of \(\sigma\), the nature of entry costs matters immensely for the welfare impact of changes in production technology \(A_y\).

The entry technology also influences the welfare impact of policies affecting the level of the population or allocative efficiency. As in Melitz (2003), increasing the population is like an extreme trade liberalization going from autarky to frictionless trade between countries. In this case, the overall welfare effect is

\[
\frac{\partial \ln w}{\partial \ln L} = \frac{1}{\sigma - 1} \left( 1 + \frac{1 - \lambda}{\sigma - 1 - (1 - \lambda)} \right)
\]

Again, at \(\sigma = 4\) the amplification through variety expansion is 50% when \(\lambda = 0\) and 0% when \(\lambda = 1\).

Now, it is plausible that different production technologies have intrinsically different setup costs.\(^{16}\) Suppose that, in the previous model, the entry technology parameter in (6) is related to the production technology by

\[
\ln A_e = -\mu \ln A_y + \epsilon
\]

where \(\epsilon\) is a component unrelated to \(A_y\) and \(\mu\) captures how fast entry costs rise with production technology (for a given cost of labor). In this case we still have

\[
\frac{\partial \ln w}{\partial \ln A_y} = 1 + \frac{1}{\sigma - 1} \frac{\partial \ln M}{\partial \ln A_y}.
\]

\(^{16}\)See Cole et al. (2015) for a model that has entry costs rising with the level of technology.
But now
\[
\frac{\partial \ln M}{\partial \ln A_y} = (1 - \lambda) \frac{\partial \ln w}{\partial \ln A_y} + \frac{\partial \ln A_e}{\partial \ln A_y} = (1 - \lambda) \frac{\partial \ln w}{\partial \ln A_y} - \mu.
\]

The welfare impact of a change in the production technology becomes
\[
\frac{\partial \ln w}{\partial \ln A_y} = 1 + \frac{1 - \lambda - \mu}{\sigma - 1 - (1 - \lambda)}.
\]

Thus, when entry costs rise with productivity, either through higher labor costs (\(\lambda\) close to 1) or higher costs of setting up more sophisticated businesses (large positive \(\mu\)), the impact of \(A_y\) on variety and welfare is dampened.

### 4.2. Span-of-Control

The entry technology matters for welfare even in a Lucas span-of-control model in which there is no love-of-variety. Consider the environment
\[
Y = \sum_{i=1}^{M} Y_i
\]
\[
Y_i = A_y L_i^\gamma
\]
\[
M = A_y^{-\mu} Y^1 - \lambda L_e^\lambda
\]

The first equation says aggregate output is the simple sum of firm output levels. The second equation specifies diminishing returns to production labor for each firm (\(\gamma < 1\)). The third equation is the technology for entry. Whereas Lucas (1978) specified overhead costs due to a single manager’s time, we allow for the possibility that overhead involves goods as well as labor. Bloom et al. (2013) for example, argue that overhead costs include some information technology equipment. Variable profits are then
\[ \pi_i = (1 - \gamma)Y_i = A_y^{\frac{1}{1-\gamma}} \left( \frac{\gamma}{w} \right)^{\frac{\gamma}{1-\gamma}}. \]

As in the love-of-variety model, free entry implies
\[ \pi_i = c_e \propto A_y^\mu w^\lambda. \]

In general equilibrium
\[ \ln w = \frac{1}{1-\gamma} - \mu \frac{1}{1-\gamma - (1-\lambda)} \ln A_y + \text{constant} \]

The welfare impact of a change in \( A_y \) here is the same as in the love-of-variety model when \( 1 - \gamma = \frac{1}{\sigma-1} \). If better production technology boosts entry, then production labor is spread more thinly across firms, limiting scale diseconomies. Thus entry can amplify the welfare impact of better technology, just as in the love-of-variety model. Unlike in the love-of-variety model, however, changes in \( L \) do not affect welfare. A bigger population increases the number of firms proportionately, but leaves aggregate productivity unchanged.

To recap, the entry technology (parameterized by \( \lambda \) and \( \mu \)) matters for welfare analysis in the span-of-control model.

4.3. Growth with Quality Ladders and Expanding Varieties

Consider a sequence of one-shot-economies, as in the love-of-variety model, with the following modifications: 1) knowledge spillovers from period \( t - 1 \) to \( t \); and 2) each entrant chooses its quality (process efficiency) \( A_t \) and the number of varieties \( v_t \) it will produce.

In each period \( t \), the past pool of knowledge \( A_{t-1} \) improves the current entry technology:
\[ c_t^e \propto e^{\mu \pi_{t-1}^{A_t}} f(v_t, A_t) w_t^\lambda = \frac{w_t^\lambda}{A_t^\lambda} \]

where \( c_t^e \) is the entry cost for a firm. An entering firm chooses its quality level \( A_t \).
and the number of varieties to produce \( v_t \). Profit maximization and free entry imply that

\[
\frac{\partial \ln \pi_t(A_t, v_t)}{\partial \ln A_t} = \frac{\partial \ln c_t^e}{\partial \ln A_t}
\]

and

\[
\frac{\partial \ln \pi_t(A_t, v_t)}{\partial \ln v_t} = \frac{\partial \ln c_t^e}{\partial \ln v_t}.
\]

Variable profits are \( \pi_t(A_t, v_t) = \pi_t A_t^{\sigma - 1} v_t \), so the firm’s optimal choice of \( A_t \) satisfies

\[
\sigma - 1 = \mu \frac{A_t}{A_{t-1}} + \frac{f_A(v_t, A_t)}{f(v_t, A_t)} A_t
\]

and its optimal choice of \( v_t \) is given by

\[
1 = \frac{f(v_t, A_t)}{f(v_t, A_t)} v_t.
\]

Assume

\[
f(v, A) = e^{v_A}, \rho > 1
\]

so that the marginal cost of producing an additional variety in a firm is increasing in the number of varieties produced in the firm, and choosing a higher technology level lowers the overall cost of producing varieties in a firm. This particular functional form implies that the growth rate of quality between \( t - 1 \) and \( t \) is

\[
g_t^A := \ln \frac{A_t}{A_{t-1}} = \ln \frac{\sigma - 1 + \frac{1}{\rho}}{\mu}
\]

and the number of varieties per firm grows at

\[
g_t^v := \ln \frac{v_t}{v_{t-1}} = \frac{1}{\rho} g_t^A
\]

The equilibrium number of firms per worker is

\[
\ln \frac{N_t}{L_t} = (1 - \lambda) \ln \frac{Y_t}{L_t} - \ln f(v_t, A_t) + \text{constant}
\]

\footnote{We want to allow higher quality to facilitate growing variety per firm because there is evidence of variety growth in the U.S. See Bernard et al. (2010) and Broda and Weinstein (2010).}
where $N_t$ is the number of firms. The number of varieties produced in the economy is $M_t := N_t v_t$. The real wage and hence welfare in this economy is

$$\ln w_t = \frac{\sigma - 1}{\sigma - 1 - (1 - \lambda)} \left( \ln A_t + \frac{\ln L_t v_t - \ln f(v_t, A_t)}{\sigma - 1} \right) + \text{constant}$$

and the growth rate of the real wage is

$$g_t^w := \frac{g^L + g^A(\sigma - 1) + g^v}{\sigma - 1 - (1 - \lambda)}.$$

Similar to the static love-of-variety model, a higher $\lambda$ implies a smaller welfare effect of changes in the level and growth rate of $A_t$ and $L_t$. This model illustrates that amplification through entry can occur in an endogenous growth model with rising quality, expanding variety, and population growth — and in which firms produce multiple varieties. In particular, amplification is from variety expansion through an increase in the number of firms, whether or not there are multiple or even growing varieties per firm.

### 4.4. The entry cost explanation

For the question posed in the title, empirical elasticities are enough. But for calibrating models it is useful to estimate $1 - \lambda - \mu$ itself. In this section, we show what our facts imply for $1 - \lambda - \mu$ in the context of the simplified Melitz model from Section 4.1.

Assuming $A_e = A^y e^e$ and using $\ln Y/M$ (value added per plant) to proxy for entry costs, the following relationship holds between value added per plant, value added per worker, and workers:

$$\ln \frac{Y}{M} = \text{constant} + \left[ 1 - \frac{(1 - \lambda - \mu) (\sigma - 1)}{\sigma - 1 - \mu} \right] \ln \frac{Y}{L} + \left[ 1 - \frac{\sigma - 1}{\sigma - 1 - \mu} \right] \ln L - \frac{\sigma - 1}{\sigma - 1 - \mu} e.$$
When $\mu = 0$, this equation is similar to regressions we ran in the previous section, but with $\ln L$ as an added control. Note, however, that $Y/L$ is endogenous to $\epsilon$ in this simple model. Years with higher $\epsilon$ (lower entry costs) should have more variety and therefore higher labor productivity. As a result, the coefficients we obtained in the previous section’s OLS regressions should not generate consistent estimates even if this simple model perfectly described the data. One can deal with this endogeneity issue if instruments are available.

To illustrate the potential magnitude of OLS endogeneity bias, suppose that $\epsilon \perp \ln A_y$ and $\epsilon \perp \ln L$. Table 2 displays the results of GMM estimation using these moment restrictions on the 1947–2012 data for the U.S. The first row of Table 2 shows that $\lambda \leq 1$ is binding.\(^\text{18}\) The implication is that if entry requires only labor ($\lambda = 1$), then the labor required for entry shrinks with better production technology ($\mu < 0$).

<table>
<thead>
<tr>
<th>$\epsilon \perp \ln A_y, \epsilon \perp \ln L$</th>
<th>$\hat{\lambda}$</th>
<th>$\hat{\mu}$</th>
<th>Amplification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon \perp \ln A_y, \epsilon \perp \ln L$</td>
<td>1</td>
<td>-0.264</td>
<td>0.088</td>
</tr>
<tr>
<td>$\epsilon \perp \ln A_y, \mu = 0$</td>
<td>0.689</td>
<td>0</td>
<td>0.116</td>
</tr>
<tr>
<td>$\epsilon \perp \ln A_y, \lambda = 0$</td>
<td>0</td>
<td>0.720</td>
<td>0.140</td>
</tr>
</tbody>
</table>

Note: Amplification is $\frac{1-\hat{\lambda}-\hat{\mu}}{\sigma-1-(1-\lambda)}$ with $\sigma = 4$.

When we impose $\mu = 0$ in the second row, there is only one parameter to

\(^\text{18}\)We impose $\lambda \in [0, 1]$ so that the share of labor and good in entry are each nonnegative.
estimate \((\lambda)\), so we relax one of the moment restrictions. The assumption that 
\(\epsilon \perp \ln A_y\) is probably the most defensible, as \(\mu > 0\) should incorporate the sys-
tematic relationship between the entry technology and the production technol-
ogy. We therefore no longer assume \(\ln L\) is orthogonal to \(\epsilon\) in the second row. In 
this case, Table 2 shows that we estimate a \(\hat{\lambda} = 0.7\). Just like the OLS regres-
sion, this IV estimate suggests that entry costs are more labor-intensive than 
goods-intensive. The corollary is that entry costs rise with development.

We can alternatively impose \(\lambda = 0\) so that entry requires goods not labor, and 
see if this forces better technology to be more costly to set up \((\mu > 0)\). Indeed, 
in the third row of Table 2 we estimate \(\hat{\mu} = 0.7\).

In all cases, the estimates of \(\lambda\) and \(\mu\) imply modest entry expansion in re-
response to better production technology — on the order of 9% to 14%, com-
pared to the 50% one would have obtained with both \(\lambda = 0\) and \(\mu = 0\). Entry 
costs must rise with the production technology through some combination of 
more expensive labor \((\lambda > 0)\) or requiring more goods \((\mu > 0)\).

5. Conclusion

In manufacturing in the U.S., India and China in recent decades, we estimate 
that the average size of businesses grow as output per worker rises. This is true 
for lifetime discounted profits in the U.S., or for average profits or size (of en-
tering businesses or all businesses) in the three countries.

These facts can be explained by a model in which entry costs rise with labor 
productivity. Entry costs can rise with productivity for multiple reasons. First, if 
entry is labor-intensive then higher wages that go along with higher labor pro-
ductivity raise the cost of entry. Second, the costs of setting up operations could 
be increasing with the level of technology, worker skill, or physical capital per 
worker. We leave it for future research to try to distinguish between these expla-
nations.

We draw several implications for policy and modeling. First, policies that
boost productivity need not increase the number of firms or plants. Second, if the choice is between denominated entry costs in terms of labor or output, the more realistic choice is denominating entry costs in terms of labor. Third, we empirically corroborate the common assumption in endogenous growth models that the cost of innovation rises with the level of technology attained.

A Details of Empirical Analysis

A1. Estimating variation in markups

Denote the markup of price over marginal cost as \( \mu := \frac{p}{c} \). We are interested in how the markup changed over time (more precisely, with aggregate manufacturing productivity) so that we can infer how profits changed over time from revenue data. Here we describe the three methods we used to infer the change in markups.

A1.1. Method 1: firm level inverse revenue share

Assume there is some input \( X \) that is not subject to adjustment costs or fixed costs. From static cost minimization alone, one can express marginal cost as

\[
c = \frac{p_x}{\partial Y(X, K)/\partial X}
\]

where \( p_x \) is the price of the input, \( K \) is a vector of all other inputs, and \( Y(X, K) \) is the production function. We can then express the markup as

\[
\mu := \frac{p_y}{c} = \frac{p_y}{p_x} \frac{\partial Y(X, K)}{\partial X} = \frac{p_y Y(X, K)}{p_x X} \frac{\partial \ln Y(X, K)}{\partial \ln X} := \frac{\alpha_x}{\omega_x}
\]

where \( \alpha_x \) is the output elasticity of input \( X \) and \( \omega_x \) is spending on \( X \) relative to revenue.

If one assumes that the production elasticity is constant for each firm over time and the proportional change in the markup over time is the same across
all firms, then the common change in markups between periods \( s \) and \( t \) is equal to the change in the revenue over cost ratio of \( X \) for every firm:

\[
\frac{\mu_t}{\mu_s} = \frac{\omega_{x,s}^f}{\omega_{x,t}^f}
\]

Of course, in the data firms do not all exhibit the same change in spending share for input \( X \). But suppose the observed change in revenue share is the true change plus classical measurement error:

\[
\begin{aligned}
\left( \frac{\omega_{x,s}^f}{\omega_{x,t}^f} \right) & = \frac{\omega_{x,s}^f}{\omega_{x,t}^f} + \epsilon, \\
& \quad \epsilon \perp \frac{\omega_{x,s}^f}{\omega_{x,t}^f}, \quad \mathbb{E}(\epsilon) = 0.
\end{aligned}
\]

Under this assumption, a consistent estimate of the change in the markup is the cross-sectional average of the change in the inverse input shares:

\[
\left( \frac{\mu_t}{\mu_s} \right) = \frac{1}{N_{s,t}} \sum_{f=1}^{N_{s,t}} \left( \frac{\omega_{x,s}^f}{\omega_{x,t}^f} \right)
\]

where \( N_{s,t} \) is the number of firms that survived between \( s \) and \( t \). Because of entry and exit, \( N_{s,t} \) is small if the lag between \( s \) and \( t \) is large. So instead of using this formula directly to calculate long differences (say between 1967 and 2012), we assume attrition and addition is independent of the measurement error and use the cumulative product of year-over-year changes. Specifically:

\[
\left( \frac{\mu_t}{\mu_s} \right) = \prod_{k=s}^{t-1} \frac{1}{N_{k,k+1}} \sum_{f=1}^{N_{k,k+1}} \left( \frac{\omega_{x,k}^f}{\omega_{x,k+1}^f} \right).
\]

As mentioned in the main text, we use intermediate inputs for \( X \) because they are less subject to adjustment and overhead costs than factors like capital and labor. Table 3 shows the mapping of model to data for U.S.
ENTRY COSTS RISE WITH DEVELOPMENT

<table>
<thead>
<tr>
<th>$p_y Y$</th>
<th>Total value of shipments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_x X$</td>
<td>Cost of materials</td>
</tr>
</tbody>
</table>

Table 3: Variables used for estimating markups, U.S.

A1.2. **Method 2: change in aggregate inverse revenue shares**

Method 1 has the disadvantage that the resulting aggregate markup variation is very sensitive to a few firms that report very low (but variable) levels of intermediate inputs. Also, since the U.S. Manufacturing Census is every five years, we drop many young firms because we can only use firms that survive for at least five years. An alternative approach that avoids these problems is to assume the distribution of markups across firms is the same over time. Then we can use the change in aggregate shares to infer the change in markups instead of using the change in firm-level shares. That is, suppose

$$\hat{\mu}_{ft} = \mu_t F E_f + \epsilon_{ft}, \quad \epsilon_{ft} \perp F E_f, \mu_t, \quad \mathbb{E}(\epsilon_{ft}) = 0.$$  

$FE_f$ is the firm fixed effect, which could reflect differences in input elasticities or multiplicative measurement error. We can run an OLS regression of firm inverse revenue share levels on year dummies to consistently estimate $\mathbb{E}_{ft}(\mu_t F E_f)$. Further, if we assume the distribution of firm fixed effects is the same over time so that $\mathbb{E}_{ft}(\mu_t F E_f) = \mu_t \mathbb{E}(F E_f)$, then we can take the ratio of the year dummies to consistently estimate the change in the aggregate markup $\mu_t$.

A1.3. **Method 3: change in median inverse revenue shares**

Method 2 is still sensitive to a few firms reporting close to zero inputs and positive revenue. Hence, we also tried using changes in the median inverse revenue share to infer the change in the markup. This is the one reported in the main text.
B Derivations for the love-for-variety model

This section lays out the intermediate steps we used for our welfare analysis and estimation in the love-for-variety model in Section 4.

B1. Welfare analysis

Endowment: $L$ units of labor

Technology: $A_y, A_e$ are exogenous.

\[ y_i = A_y l_i \quad \text{(Intermediate goods production)} \]
\[ Y = \left[ \int_M^{\sigma -1} \frac{y_i}{\sigma} \, di \right]^\frac{\sigma}{\sigma -1} \quad \text{(Final goods production)} \]
\[ M = A_e L_e Y_e^{1-\lambda} \quad \text{(Entry goods production)} \]

Household’s problem:

\[ \max_C u \left( \frac{C}{L} \right), \quad s.t. C \leq wL + \pi M - c_e M \]

Firm’s problem:

\[ \max_{\{y_i\}} Y - \sum p_i y_i, \quad s.t. Y \leq \left[ \int_M^{\sigma -1} \frac{y_i}{\sigma} \, di \right]^\frac{\sigma}{\sigma -1} \quad \text{(Final goods producer)} \]
\[ \max_{y_i, l_i} p_i y_i - w l_i, \quad s.t. y_i \leq A_y l_i, \quad y_i = p_i^{\sigma} Y \quad \text{(Intermediate goods producer)} \]
\[ \max_{Y_e, L_e} c_e M - Y_e - w L_e, \quad s.t. M \leq A_e L_e Y_e^{1-\lambda} \quad \text{(Entry goods producer)} \]

Zero-profit-condition:

\[ c_e = \pi \]

Market clearing conditions:

\[ L = L_e + L_M \]
ENTRY COSTS RISE WITH DEVELOPMENT

\[ Y = C + Y_e \]

Solving the intermediate goods producer’s problem, we have

\[ p_i = \frac{w}{A_y} \frac{\sigma - 1}{\sigma}, \quad \pi_i = \frac{w l_i}{\sigma - 1} = \frac{p_i y_i}{\sigma} \]

Also, solving the entry goods producer’s problem yields

\[ c_e = \frac{1}{A_e} \left( \frac{w}{\lambda} \right)^{\lambda} \left( \frac{1}{1 - \lambda} \right)^{1-\lambda} \]

Using these solutions, the labor market clearing condition and the zero-profit-condition, we get

\[ \frac{L_m}{L} = \frac{w L_m}{w L_m + w L_e} = \frac{(\sigma - 1) \pi_i}{(\sigma - 1) \pi_i + \lambda \pi_i} = \frac{\sigma - 1}{\sigma - 1 + \lambda} \]

As a corollary,

\[ \frac{L_e}{L} = \frac{\lambda}{\sigma - 1 + \lambda} \]

Substituting the solutions for \( L_m \) into the final goods production function, the relationship between \( \pi_i \) and \( w l_i \), the entry goods production function and price of entry goods, we get the following simultaneous equations that express \( Y/L, w, M, \) and \( c_e \) in terms of exogenous variables.

\[
Y = A_y L_m M^{\frac{1}{\sigma - 1}} = \frac{\sigma - 1}{\sigma - 1 + \lambda} A_y L M^{\frac{1}{\sigma - 1}}
\]

\[
w = \frac{\sigma - 1}{\sigma} M p_i y_i = \frac{\sigma - 1 + \lambda}{\sigma} \frac{Y}{L}
\]

\[
M c_e = \frac{w L_e}{\lambda} = \frac{w L}{\sigma - 1 + \lambda}
\]

\[
c_e = \frac{1}{A_e} \left( \frac{w}{\lambda} \right)^{\lambda} \left( \frac{1}{1 - \lambda} \right)^{1-\lambda}
\]

Rearranging and expressing in natural logs, we have the following simultaneous
equations that relates $w$, $M$ and $c_e$ to the exogenous variables.

\[
\ln M + \ln \frac{c_e}{w} = \ln L - \ln(\sigma - 1 + \lambda) =: b_{\text{pop}} \\
\ln w - \frac{1}{\sigma - 1} \ln M = \ln \frac{\sigma - 1}{\sigma} + \ln A_y =: b_{\text{tech}} \\
\lambda \ln w - \ln c_e = \ln A_e + \lambda \ln \lambda + (1 - \lambda) \ln(1 - \lambda) =: b_{\text{entry}}
\]

Solving these gives the following equations for the endogenous variables in terms of the exogenous variables.

\[
\ln w = \frac{b_{\text{pop}} + b_{\text{entry}} + (\sigma - 1)b_{\text{tech}}}{\sigma - 1 - (1 - \lambda)} \\
\ln M = (\sigma - 1)(\ln w - b_{\text{tech}}) = (\sigma - 1) \frac{b_{\text{pop}} + b_{\text{entry}} + (1 - \lambda)b_{\text{tech}}}{\sigma - 1 - (1 - \lambda)} \\
\ln c_e = \lambda \ln w - b_{\text{entry}} = \frac{\lambda b_{\text{pop}} - (\sigma - 2)b_{\text{entry}} + \lambda(\sigma - 1)b_{\text{tech}}}{\sigma - 1 - (1 - \lambda)}
\]

The welfare analysis in Section 4 follows directly from these three equations. The first equation gives the welfare impact of changes in productivity and population. The second equation illustrates the variety expansion channel. The number of varieties $M$ responds to changes in production productivity only if the goods share of entry is positive. Finally, the last equation shows the entry costs rise with exogenous productivity and population only if the labor share of entry is positive.
**B2. Estimation**

Next we show the intermediate steps to deriving the objective functions in our estimation procedure. As in the main text, we introduce an idiosyncratic component to entry technology in the form of $A_e = A_y^{-\mu}e^\epsilon$. The exogenous variables are related to the observables by

\[ b_{pop} = \ln L - \ln(\sigma - 1 + \lambda) \]  \hspace{1cm} (10)
\[ b_{tech} = \ln \frac{Y \sigma - 1 + \lambda}{\sigma} - \frac{1}{\sigma - 1} \ln M \]  \hspace{1cm} (11)
\[ b_{entry} = \lambda \ln \frac{Y \sigma - 1 + \lambda}{\sigma} - \ln c_e \]  \hspace{1cm} (12)

Substituting $A_e = A_y^{-\mu}e^\epsilon$ and the definition of $b_{tech}$ into the definition of $b_{entry}$, we have

\[ b_{entry} = -\mu \left( b_{tech} - \ln \frac{\sigma - 1}{\sigma} \right) + \epsilon + \lambda \ln \lambda + (1 - \lambda) \ln(1 - \lambda) \]

and

\[ \epsilon = \lambda \ln \frac{Y \sigma - 1 + \lambda}{\sigma} - \ln c_e + \mu \left( b_{tech} - \ln \frac{\sigma - 1}{\sigma} \right) - \lambda \ln \lambda - (1 - \lambda) \ln(1 - \lambda) \]
\[ = (\lambda + \mu) \ln \frac{Y \sigma - 1 + \lambda}{\sigma} - \ln c_e - \frac{\mu}{\sigma - 1} \ln M \]
\[ - \mu \ln \frac{\sigma - 1}{\sigma} - \lambda \ln \lambda - (1 - \lambda) \ln(1 - \lambda) \]

So if $\lambda = 0$ and $\mu = 0$, periods of high entry costs ($c_e$) are due to low idiosyncratic entry goods production efficiency ($\epsilon$).

Assuming $\epsilon_t$ is independent of $\ln A_{y,t}$ and $\ln L_t$, then we have

\[ \mathbb{E}[(\epsilon_t - \mathbb{E}\epsilon_t) \ln L_t] = 0, \quad \mathbb{E}[(\epsilon_t - \mathbb{E}\epsilon_t) \ln A_{y,t}] = 0 \]
Since
\[ \ln \frac{Y}{L_t} - \frac{1}{\sigma - 1} \ln M_t = \ln A_{y,t} + \ln \frac{\sigma}{\sigma - 1 + \lambda} - \ln \frac{\sigma - 1}{\sigma} \]
we have the following identifying restrictions for \( \lambda \) and \( \mu \)
\[ \mathbb{E} g_t = 0, \quad g_t := \begin{bmatrix} g^1_t \\ \tilde{e}_t (\ln \frac{Y}{L_t} - \frac{1}{\sigma - 1} \ln M_t) \\ g^2_t \end{bmatrix} = \begin{bmatrix} \tilde{e}_t (\ln \frac{Y}{L_t} - \frac{1}{\sigma - 1} \ln M_t) \\ \tilde{e}_t \ln L_t \end{bmatrix} \]
where
\[ \tilde{e}_t := \epsilon - \mathbb{E} \epsilon = (\lambda + \mu) \ln \frac{Y}{L} - \ln c_e - \frac{\mu}{\sigma - 1} \ln M \]
The tilde notation denotes the deviation of a variable from its expected value. We construct the sample analogue of \( \tilde{e} \) by using the deviation from the sample mean for \( \ln \frac{Y}{L}, \ln c_e \) and \( \ln M \).

The GMM estimator of \( \lambda \) and \( \mu \) is found by choosing \( \lambda, \mu \) to solve \( \frac{1}{T} g_t(\lambda, \mu) = 0 \). Since the loss function is linear in \( \lambda \) and \( \mu \), we have the following close-form solution:
\[
\begin{bmatrix} \hat{\lambda} \\ \hat{\mu} \end{bmatrix} = \left[ \begin{array}{cc} \hat{\text{Cov}} (\ln \frac{Y}{L_t}, \ln \frac{Y}{L_t} - \frac{1}{\sigma - 1} \ln M_t) & \hat{\text{Var}} (\ln \frac{Y}{L_t} - \frac{\ln M_t}{\sigma - 1}) \\ \hat{\text{Cov}} (\ln \frac{Y}{L_t}, \ln L_t) & \hat{\text{Cov}} (\ln \frac{Y}{L_t} - \frac{\ln M_t}{\sigma - 1}, \ln L_t) \end{array} \right]^{-1} \times \left[ \begin{array}{c} \hat{\text{Cov}} (\ln c_{e,t}, \ln \frac{Y}{L_t} - \frac{1}{\sigma - 1} \ln M_t) \\ \hat{\text{Cov}} (\ln c_{e,t}, \ln L_t) \end{array} \right]
\]

We calculate the standard errors for the estimate using the asymptotic variance
\[ \frac{1}{N} \left[ \hat{G} \hat{S}^{-1} \hat{G} \right]^{-1} = \frac{1}{N} \hat{G}^{-1} \hat{S} \hat{G}^{-1} \]
where
\[
\hat{S} := \frac{1}{T} \sum_{t=1}^{T} \hat{g}_t(\hat{\lambda}, \hat{\mu}) \hat{g}_t(\hat{\lambda}, \hat{\mu})', \quad \hat{G} := \frac{1}{T} \sum_{t=1}^{T} \begin{bmatrix} \frac{\partial g_1}{\partial \lambda} & \frac{\partial g_2}{\partial \lambda} \\
\frac{\partial g_1}{\partial \mu} & \frac{\partial g_2}{\partial \mu} \end{bmatrix}
\]

Both \( \hat{S} \) and \( \hat{G} \) are evaluated at the estimates of \( \lambda \) and \( \mu \).

**References**


